RELATIVE HUMIDITY: SEEING YOUR BREATH

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 5.43.

The vapour pressure equation gives the phase boundary curve between the liquid and gas phases of a substance if we can assume that the gas is an ideal gas:

$$P_v = Ke^{-L/RT} \tag{1}$$

where K is a constant and L is the latent heat of vapourization. We can use this equation to get a rough solution to an everyday situation. When you're outdoors in cold weather, you can frequently see your breath, which is caused by the humidity in your breath condensing in the colder air because the combination of the vapour pressure in your breath and the outside air exceeds the equilibrium pressure for the ambient temperature.

To get a rough idea how this works, we'll throw in some numbers. Suppose the breath you exhale has a temperature of 35° C (normal body temperature is 37°C) and 90% humidity, and the outside air has a temperature of 10° C, with an unknown humidity that you wish to find.

To deal with this situation, we can use the value of K that we found earlier, which is based on the data in Schroeder's Figure 5.11 over a temperature range of 0° C to 40° C. This gives $K = 1.63 \times 10^{11}$ Pa. Also, $L = 43.99 \times 10^3$ J mol^{-1} and the gas constant is $R = 8.314$ in SI units.

The equilibrium pressure at 35° C $= 308$ K is then

$$
\begin{aligned}
P_v(308) &= \left(1.63 \times 10^{11}\right) e^{-43.99 \times 10^3/(8.314)(308)} \tag{2} \\
&= 5643 \text{ Pa} \tag{3}
\end{aligned}
$$

The vapour pressure in your exhaled breath at 90% humidity is therefore

$$P = 0.9 \times 5643 = 5079 \text{ Pa} \tag{4}$$

The temperature of the gas just outside your nose after you exhale will be some value between 35° C and 10° C. Suppose we just take the average (which would occur if you mix equal volumes of the two gases) and say that the temperature of the air just outside your nose is

$$T_{nose} = \frac{283 + 308}{2} = 295.5 \text{ K} \tag{5}$$

The equilibrium vapour pressure at this temperature can be calculated from 1:

$$P_v(295.5) = 2729 \text{ Pa} \tag{6}$$

If you can see your breath, the vapour pressure outside your nose must be at least this much. If we're also assuming that the vapour pressure is the average of the pressure in the air and the vapour pressure that you exhale, then the vapour pressure P_{air} of the air must satisfy

$$\frac{5079 + P_{air}}{2} \geq 2729 \tag{7}$$

This gives

$$P_{air} \geq 378 \text{ Pa} \tag{8}$$

To calculate the relative humidity of the outside air, we need the equilibrium pressure at $10°$ C $= 283$ K which we can again get from 1:

$$P_v(283) = 1237 \text{ Pa} \tag{9}$$

Therefore, the humidity of the outside air is at least

$$\frac{378}{1237} = 0.306 \tag{10}$$

so we'd expect the humidity to be greater than around 30%.

I'm not sure how accurate is the assumption that we can just take the averages of temperature and vapour pressure in the gas into which you exhale, but it shouldn't be too far off.

One interesting fact comes out of this analysis. It's not necessary for the air to be *really* cold in order for you to see your breath. All that's needed is for the outside air to be just slightly cooler than your breath and have a high enough humidity. You can sometimes see this effect on days when the air is exceptionally humid (for example, during rainy weather) even if it's not particularly cold.