CLOUD FORMATION

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Reference: Daniel V. Schroeder, *An Introduction to Thermal Physics*, (Addison-Wesley, 2000) - Problem 5.44.

As warm, moist air at the Earth's surface rises, it expands and cools until the vapour pressure reaches the dew point at a certain height. When the air reaches that height, the vapour starts to condense and form clouds. We can get a rough estimate of the height at which clouds should form by combining a few of our earlier results.

First, we'll assume that the rising air expands adiabatically and that the temperature gradient in the atmosphere is at the adiabatic lapse rate. This rate was calculated earlier to be

$$\frac{dT}{dz}=-0.00976\text{ deg m}^{-1} \tag{1}$$

This is the rate at which the temperature decreases as we go higher in the atmosphere. Since this rate is a constant, the temperature as a function of height z above the Earth's surface is

$$T(z)=T_0-0.00976z \tag{2}$$

where T_0 is the temperature at the surface.

Secondly, if we assume that the atmosphere is just at the critical point where convection begins, then its pressure still obeys the barometric equation:

$$P(z)=P_0e^{-mgz/kT} \tag{3}$$

where P_0 is the pressure at the surface. The values we used earlier are $m=4.81\times10^{-26}$ kg for the mass of a typical air molecule, $g=9.8\text{ m s}^{-2}$ and $k=1.38\times10^{-23}$ in SI units.

Finally, we can use the vapour pressure equation which gives the vapour pressure at which an ideal gas condenses into liquid:

$$P_v(T)=Ke^{-L/RT} \tag{4}$$

where $K = 1.63 \times 10^{11}$ Pa is a constant, $L = 4.399 \times 10^4$ is the latent heat of vapourization and the gas constant is $R = 8.314$, both in SI units. Using 2 we can write this as a function of z:

$$P_v(z) = Ke^{-L/R(T_0 - 0.00976z)} \tag{5}$$

To apply all this to cloud formation, we'll throw in some specific numbers. Let's assume that the surface temperature is $T_0 = 25^\circ$ C $= 298$ K and the surface humidity is 50%. This gives a vapour pressure at $z = 0$ of

$$P_v(0) = 0.5Ke^{-L/R\times 298} = 1585 \text{ Pa} \tag{6}$$

If we use this as P_0 in 3 (that is, we're assuming that the vapour pressure follows the same exponential relation as the overall pressure), and take the temperature as given by 2, then we have an equation for the partial pressure of water vapour as a function of height z.

$$P(z) = P_0 e^{-mgz/k(T_0 - 0.00976z)} \tag{7}$$

The height at which clouds will form is the value of z such that $P(z) = P_v(z)$, since then the actual vapour pressure is equal to the vapour pressure where condensation occurs. That is, we want z such that

$$P_0 e^{-mgz/k(T_0 - 0.00976z)} = Ke^{-L/R(T_0 - 0.00976z)} \tag{8}$$

Although this equation may look scary, it's actually quite easy to solve for z by taking the logs:

$$\ln\frac{P_0}{K} - \frac{mgz}{k(T_0 - 0.00976z)} = -\frac{L}{R(T_0 - 0.00976z)} \tag{9}$$

Rearranging, we get

$$(T_0 - 0.00976z)\ln\frac{P_0}{K} = \frac{mgz}{k} - \frac{L}{R} \tag{10}$$

$$z = \left[T_0 \ln\frac{P_0}{K} + \frac{L}{R}\right]\left[0.00976\ln\frac{P_0}{K} + \frac{mg}{k}\right]^{-1} \tag{11}$$

Plugging in the numbers, we get

$$z = 1416 \text{ m} \tag{12}$$

Thus we'd expect clouds to form at around 1.4 km above the surface.

PINGBACKS

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