HERMITIAN OPERATORS - A FEW EXAMPLES

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Here are a few more results about hermitian operators:

Suppose we are given two hermitian operators $\Omega$ and $\Lambda$. We’ll look at some combinations of these operators.

The operator $\Omega\Lambda$ has the hermitian conjugate

$$ (\Omega\Lambda)^\dagger = \Lambda^\dagger \Omega^\dagger = \Lambda \Omega $$

Thus the product operator $\Omega\Lambda$ is hermitian only if $\Lambda$ and $\Omega$ commute.

The operator $\Omega\Lambda + \lambda \Omega$ for some complex scalar $\lambda$ has the hermitian conjugate

$$ (\Omega\Lambda + \lambda \Omega)^\dagger = \Lambda^\dagger \Omega^\dagger + \lambda^* \Omega^\dagger $$

$$ = \Lambda \Omega + \lambda^* \Omega $$

This operator is therefore hermitian only if $\Lambda$ and $\Omega$ commute and $\lambda$ is real.

The commutator has the hermitian conjugate

$$ [\Omega, \Lambda]^\dagger = (\Omega\Lambda - \Lambda\Omega)^\dagger $$

$$ = \Lambda \Omega - \Omega \Lambda $$

$$ = [\Lambda, \Omega] $$

Thus the commutator is anti-hermitian (the hermitian conjugate is the negative of the original operator).

Finally, what happens if we multiply the commutator by $i$?

$$ (i[\Omega, \Lambda])^\dagger = -i (\Omega\Lambda - \Lambda\Omega)^\dagger $$

$$ = -i (\Lambda \Omega - \Omega \Lambda) $$

$$ = -i [\Lambda, \Omega] $$

$$ = i [\Omega, \Lambda] $$
Thus the operator $i [\Omega, \Lambda]$ is hermitian.