

HERMITIAN OPERATORS - A FEW EXAMPLES

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog.

References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 1.6.2.

Here are a few more results about hermitian operators.

Suppose we are given two hermitian operators Ω and Λ . We'll look at some combinations of these operators.

The operator $\Omega\Lambda$ has the hermitian conjugate

$$(\Omega\Lambda)^\dagger = \Lambda^\dagger\Omega^\dagger = \Lambda\Omega \quad (1)$$

Thus the product operator $\Omega\Lambda$ is hermitian only if Λ and Ω commute.

The operator $\Omega\Lambda + \lambda\Omega$ for some complex scalar λ has the hermitian conjugate

$$(\Omega\Lambda + \lambda\Omega)^\dagger = \Lambda^\dagger\Omega^\dagger + \lambda^*\Omega^\dagger \quad (2)$$

$$= \Lambda\Omega + \lambda^*\Omega \quad (3)$$

This operator is therefore hermitian only if Λ and Ω commute and λ is real.

The commutator has the hermitian conjugate

$$[\Omega, \Lambda]^\dagger = (\Omega\Lambda - \Lambda\Omega)^\dagger \quad (4)$$

$$= \Lambda\Omega - \Omega\Lambda \quad (5)$$

$$= [\Lambda, \Omega] \quad (6)$$

$$= -[\Omega, \Lambda] \quad (7)$$

Thus the commutator is anti-hermitian (the hermitian conjugate is the negative of the original operator).

Finally, what happens if we multiply the commutator by i ?

$$(i[\Omega, \Lambda])^\dagger = -i(\Omega\Lambda - \Lambda\Omega)^\dagger \quad (8)$$

$$= -i(\Lambda\Omega - \Omega\Lambda) \quad (9)$$

$$= -i[\Lambda, \Omega] \quad (10)$$

$$= i[\Omega, \Lambda] \quad (11)$$

Thus the operator $i[\Omega, \Lambda]$ is hermitian.