HAMILTONIAN FOR THE ELECTROMAGNETIC FORCE

Here we derive the equations of motion for the electromagnetic force using the Hamiltonian formalism.

The Hamiltonian is given by

\[ H(q,p) = \sum_i p_i \dot{q}_i - L(q,\dot{q}) \]  

where the velocities \( \dot{q}_i \) are expressed in terms of the positions \( q_i \) and momenta \( p_i \). The electromagnetic Lagrangian is

\[ L = \frac{1}{2} m \dot{v} \cdot \dot{v} - q \phi + \frac{q}{c} \dot{v} \cdot A \]  

where \( \phi \) is the electric potential and \( A \) is the magnetic potential, with \( \dot{v} \) the velocity of the charge \( q \) with mass \( m \). To convert to the Hamiltonian, we need the momentum, defined as

\[ p_i = \frac{\partial L}{\partial \dot{q}_i} \]

In this case, the generalized velocity is given by

\[ \dot{q}_i = v_i \]  

so we have

\[ p_i = m v_i + \frac{q}{c} A_i \]  

or, in vector notation

\[ p = m \dot{v} + \frac{q}{c} A \]  

\[ \dot{v} = \frac{p}{m} - \frac{q}{mc} A \]  

The Lagrangian is therefore

\[ L = \frac{|p - qA/c|^2}{2m} - q\phi + \frac{q}{c} \left( \frac{p}{m} - \frac{q}{mc} A \right) \cdot A \]  

The first sum in the Hamiltonian is

$$\sum_i p_i \dot{q}_i = \mathbf{p} \cdot \mathbf{v} = \mathbf{p} \cdot \left( \frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right)$$

(8)

The Hamiltonian is then

$$H = \mathbf{p} \cdot \left( \frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) - \frac{\left| \mathbf{p} - \frac{q}{c} \mathbf{A} \right|^2}{2m} + q\phi - \frac{q}{c} \left( \frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \cdot \mathbf{A}$$

(9)

$$= \left( \frac{\mathbf{p}}{m} - \frac{q}{mc} \mathbf{A} \right) \left( \mathbf{p} - \frac{q}{c} \mathbf{A} \right) - \frac{\left| \mathbf{p} - \frac{q}{c} \mathbf{A} \right|^2}{2m} + q\phi$$

(10)

$$= \frac{\left| \mathbf{p} - \frac{q}{c} \mathbf{A} \right|^2}{2m} + q\phi$$

(11)

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