INFINITE SQUARE WELL - FORCE TO DECREASE WELL WIDTH

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References: Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Section 5.2, Exercise 5.2.4.

One way of comparing the classical and quantum pictures of a particle in an infinite square well is to calculate the force exerted on the walls by the particle. If a particle is in state $|n\rangle$, its energy is

$$E_n = \frac{(n\pi\hbar)^2}{2mL^2} \tag{1}$$

If the particle remains in this state as the walls are slowly pushed in, so that L slowly decreases, then its energy E_n will increase, meaning that work is done on the system. The force is the change in energy per unit distance, so the force required is

$$F = -\frac{\partial E_n}{\partial L} = \frac{(n\pi\hbar)^2}{mL^3} \tag{2}$$

If we treat the system classically, then a particle with energy E_n between the walls is effectively a free particle in this region (since the potential $V = 0$ there), so all its energy is kinetic. That is

$$\begin{aligned} E_n &= \frac{1}{2}mv^2 && (3)\\ v &= \sqrt{\frac{2E_n}{m}} && (4)\\ &= \frac{n\pi\hbar}{mL} && (5) \end{aligned}$$

The classical particle bounces elastically between the two walls, which means its velocity is exactly reversed at each collision. The momentum transfer in such a collision is

$$\Delta p = 2mv = \frac{2n\pi\hbar}{L} \tag{6}$$

The time between successive collisions on the same wall is

$$\Delta t=\frac{2L}{v}=\frac{2mL^{2}}{n\pi\hbar} \tag{7}$$

Thus the average force exerted on one wall is

$$\bar{F}=\frac{\Delta p}{\Delta t}=\frac{(n\pi\hbar)^{2}}{mL^{3}} \tag{8}$$

Comparing with 2, we see that the quantum and classical forces in this case are the same.