

## FERMIONS AND BOSONS IN THE INFINITE SQUARE WELL

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 10, Exercise 10.3.4.

Suppose we have two identical particles in an infinite square well. The energy levels in a well of width  $L$  are

$$E = \frac{(\pi n \hbar)^2}{2mL^2} \quad (1)$$

where  $n = 1, 2, 3, \dots$ . The corresponding wave functions are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (2)$$

If the total energy of the two particles is  $\pi^2 \hbar^2 / mL^2$ , the only possible configuration is for both particles to be in the ground state  $n = 1$ . This means the particles must be bosons, so the state vector is

$$|x_1, x_2\rangle = \frac{2}{L} \sin \frac{\pi x_1}{L} \sin \frac{\pi x_2}{L} \quad (3)$$

If the total energy is  $5\pi^2 \hbar^2 / 2mL^2$ , then one particle is in the state  $n = 1$  and the other is in  $n = 2$ . Since the states are different, the particles can be either bosons or fermions. For bosons, the state vector is

$$|x_1, x_2\rangle = \frac{1}{\sqrt{2}} \left[ \frac{2}{L} \sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} + \frac{2}{L} \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right] \quad (4)$$

$$= \frac{\sqrt{2}}{L} \left[ \sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} + \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right] \quad (5)$$

For fermions, the state must be antisymmetric, so we have

$$|x_1, x_2\rangle = \frac{\sqrt{2}}{L} \left[ \sin \frac{\pi x_1}{L} \sin \frac{2\pi x_2}{L} - \sin \frac{2\pi x_1}{L} \sin \frac{\pi x_2}{L} \right] \quad (6)$$