COMPOUND SYSTEMS OF FERMIONS AND BOSONS

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 10, Exercise 10.3.6.

In a system of identical particles, we've seen that if the particles are bosons, the state vector is symmetric with respect to the exchange of any two particles (that is, $\psi(a,b) = \psi(b,a)$ where a and b are any two of the particles in the system), while for fermions, the state vector is antisymmetric, meaning that $\psi(a,b) = -\psi(a,b)$. What happens if we have a compound object such as a hydrogen atom that is composed of a collection of fermions and/or bosons?

Suppose we look at the hydrogen atom in particular. It is composed of a proton and an electron, both of which are fermions. The proton and electron are not, of course, identical particles, but now suppose we have *two* hydrogen atoms. The two protons *are* identical fermions, just as are the two electrons. However, when analyzing a system of two hydrogen atoms, the relevant question is what happens to the state vector if we exchange the two atoms. In doing so, we exchange both the two protons and the two electrons. Each exchange multiplies the state vector by -1, so the net effect of exchanging both protons and both electrons is to multiply the state vector by $(-1)^2 = 1$. In other words, a hydrogen atom acts as a boson, even though it is composed of two fermions.

In general, if we have a compound object containing n fermions, then the state vector for a system of two such objects is multiplied by $(-1)^n$ when these two objects are exchanged. That is, a compound object containing an even number of fermions behaves as a boson, while if it contains an odd number of fermions, it behaves as a fermion.

A compound object consisting entirely of bosons will always behave as a boson, no matter how many such bosonic particles it contains, since interchanging all n bosons just multiplies the state vector by $(+1)^n = 1$.