

## ANGULAR MOMENTUM OF CIRCULAR MOTION

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 12, Exercise 12.3.6.

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A particle of mass  $\mu$  constrained to move (at constant speed  $v$ , we assume) on a circle of radius  $a$  centred at the origin in the  $xy$  plane has a constant kinetic energy of  $\frac{1}{2}\mu v^2$ . As its momentum  $\mathbf{p}$  is always perpendicular to the radius vector  $\mathbf{r}$ , the angular momentum is given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mu av \hat{\mathbf{z}} = L_z \hat{\mathbf{z}} \quad (1)$$

The energy can thus be written as

$$H = \frac{1}{2}\mu v^2 = \frac{L_z^2}{2\mu a^2} \quad (2)$$

In polar coordinates, the angular momentum operator is

$$L_z = -i\hbar \frac{\partial}{\partial \phi} \quad (3)$$

The eigenvalue problem for this system is therefore

$$H\psi = E\psi \quad (4)$$

$$-\frac{\hbar^2}{2\mu a^2} \frac{\partial^2 \psi}{\partial \phi^2} = E\psi \quad (5)$$

The eigenvalues of  $L_z$  are the solutions of

$$-\hbar^2 \frac{\partial^2 \psi}{\partial \phi^2} = \ell_z \psi \quad (6)$$

which are

$$\psi = Ae^{i\ell_z \phi / \hbar} = Ae^{im\phi} \quad (7)$$

for some constant  $A$ , with the quantization condition (arising from the requirement that  $\psi(\phi + 2\pi) = \psi(\phi)$ )

$$\ell_z = m\hbar \quad (8)$$

where  $m$  is an integer (positive, negative or zero). Plugging this into 5 we find

$$E = \frac{\hbar^2 m^2}{2\mu a^2} \quad (9)$$

Each energy is two-fold degenerate since  $\pm m$  both give the same energy. This corresponds to the particle moving round the circle in the clockwise or counterclockwise direction.