ANGULAR MOMENTUM OF CIRCULAR MOTION

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 12, Exercise 12.3.6.

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A particle of mass μ constrained to move (at constant speed v, we assume) on a circle of radius a centred at the origin in the xy plane has a constant kinetic energy of $\frac{1}{2}\mu v^2$. As its momentum $\mathbf{p}$ is always perpendicular to the radius vector $\mathbf{r}$, the angular momentum is given by

$$\mathbf{L}=\mathbf{r}\times\mathbf{p}=\mu av\hat{\mathbf{z}}=L_z\hat{\mathbf{z}} \tag{0.1}$$

The energy can thus be written as

$$H=\frac{1}{2}\mu v^2=\frac{L_z^2}{2\mu a^2} \tag{0.2}$$

In polar coordinates, the angular momentum operator is

$$L_z=-i\hbar\frac{\partial}{\partial\phi} \tag{0.3}$$

The eigenvalue problem for this system is therefore

$$H\psi = E\psi \tag{0.4}$$

$$-\frac{\hbar^2}{2\mu a^2}\frac{\partial^2\psi}{\partial\phi^2} = E\psi \tag{0.5}$$

The eigenvalues of L_z are the solutions of

$$-\hbar^2\frac{\partial^2\psi}{\partial\phi^2} = \ell_z\psi \tag{0.6}$$

which are

$$\psi=Ae^{i\ell_z\phi/\hbar}=Ae^{im\phi} \tag{0.7}$$

for some constant A, with the quantization condition (arising from the requirement that $\psi(\phi+2\pi)=\psi(\phi)$)

$$\ell_z = m\hbar \tag{0.8}$$

where m is an integer (positive, negative or zero). Plugging this into 0.5 we find

$$E = \frac{\hbar^2 m^2}{2\mu a^2} \tag{0.9}$$

Each energy is two-fold degenerate since $\pm m$ both give the same energy. This corresponds to the particle moving round the circle in the clockwise or counterclockwise direction.