HYDROGEN ATOM: RADIAL FUNCTION AT LARGE R

When solving the 3-d Schrödinger equation for a spherically symmetric potential, the radial function has the asymptotic form for large \( r \), and for the energy \( E < 0 \):

\[
U_{El}(r) \rightarrow Ar^{±me^2/κ\hbar^2}e^{−κr}
\]  

where

\[κ \equiv \sqrt{\frac{2m|E|}{\hbar^2}}\]  

For the hydrogen atom, the function \( U_{El} \) is obtained from a series solution of the differential equation with the result

\[
U_{El} = e^{−\rho}v_{El}
\]

\[v_{El} = \rho^{l+1} \sum_{k=0}^{∞} C_k \rho^k\]

\[\rho = \sqrt{\frac{-2mE}{\hbar^2}}r = κr\]

To keep the wave function finite at large \( r \), we require the series to terminate, which leads to the quantized energy levels, given by

\[E_n = -\frac{me^4}{2\hbar^2 n^2}\]  

The series in \( v_{El} \) terminates at a value of \( k = n - l - 1 \), so the function \( v_{El} \) is a polynomial in \( ρ \), and thus in \( r \), of degree \( n \). Since the actual radial function is
we have that $R_{nl}$ is a polynomial of degree $n - 1$ in $r$ multiplied by the exponential $e^{-\rho} = e^{-\kappa r}$. That is, for large $r$

$$R_{nl} \sim r^{n-1} e^{-\kappa r}$$  \hspace{1cm} (9)

To show that this is consistent with 1, we use 7 and 2.

Comparing this with 1, we see that

$$r^n = r^{me^2/\kappa h^2}$$  \hspace{1cm} (13)

so the condition is satisfied.