

GENERAL 2X2 MATRIX IN TERMS OF PAULI MATRICES

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.3.5.

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Any 2×2 matrix can be written as a linear combination of the three Pauli matrices and the unit matrix. That is, for an arbitrary matrix M we have

$$M = \sum_{\alpha} m_{\alpha} \sigma_{\alpha} \quad (1)$$

where the coefficients are found from

$$m_{\alpha} = \frac{1}{2} \text{Tr}(M \sigma_{\alpha}) \quad (2)$$

We can write this out explicitly as follows

$$M = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \quad (3)$$

We then get

$$m_0 = \frac{1}{2} \text{Tr}(M\sigma_0) \quad (4)$$

$$= \frac{1}{2} \text{Tr}(MI) \quad (5)$$

$$= \frac{1}{2} \text{Tr}(M) \quad (6)$$

$$= \frac{\alpha + \delta}{2} \quad (7)$$

$$m_1 = \frac{1}{2} \text{Tr}(M\sigma_1) \quad (8)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \quad (9)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} \beta & \alpha \\ \delta & \gamma \end{bmatrix} \right) \quad (10)$$

$$= \frac{\beta + \gamma}{2} \quad (11)$$

$$m_2 = \frac{1}{2} \text{Tr}(M\sigma_2) \quad (12)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \right) \quad (13)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} i\beta & -i\alpha \\ i\delta & -i\gamma \end{bmatrix} \right) \quad (14)$$

$$= i \frac{\beta - \gamma}{2} \quad (15)$$

$$m_3 = \frac{1}{2} \text{Tr}(M\sigma_3) \quad (16)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right) \quad (17)$$

$$= \frac{1}{2} \text{Tr} \left(\begin{bmatrix} \alpha & -\beta \\ \gamma & -\delta \end{bmatrix} \right) \quad (18)$$

$$= \frac{\alpha - \delta}{2} \quad (19)$$

Thus, in more conventional notation

$$M = \frac{1}{2} [(\alpha + \delta)I + (\beta + \gamma)\sigma_x + i(\beta - \gamma)\sigma_y + (\alpha - \delta)\sigma_z] \quad (20)$$

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