GENERAL 2X2 MATRIX IN TERMS OF PAULI MATRICES

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.3.5.

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Any 2×2 matrix can be written as a linear combination of the three Pauli matrices and the unit matrix. That is, for an arbitrary matrix M we have

$$M=\sum_{\alpha}m_{\alpha}\sigma_{\alpha} \tag{1}$$

where the coefficients are found from

$$m_{\alpha}=\frac{1}{2}\mathrm{Tr}\left(M\sigma_{\alpha}\right) \tag{2}$$

We can write this out explicitly as follows

$$M=\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right] \tag{3}$$

We then get

$$m_0 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_0\right) \tag{4}$$
$$= \frac{1}{2}\mathrm{Tr}\left(MI\right) \tag{5}$$
$$= \frac{1}{2}\mathrm{Tr}\left(M\right) \tag{6}$$
$$= \frac{\alpha+\delta}{2} \tag{7}$$
$$m_1 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_1\right) \tag{8}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}0 & 1\\ 1 & 0\end{array}\right]\right) \tag{9}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\beta & \alpha\\ \delta & \gamma\end{array}\right]\right) \tag{10}$$
$$= \frac{\beta+\gamma}{2} \tag{11}$$
$$m_2 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_2\right) \tag{12}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}0 & -i\\ i & 0\end{array}\right]\right) \tag{13}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}i\beta & -i\alpha\\ i\delta & -i\gamma\end{array}\right]\right) \tag{14}$$
$$= i\frac{\beta-\gamma}{2} \tag{15}$$
$$m_3 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_3\right) \tag{16}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}1 & 0\\ 0 & -1\end{array}\right]\right) \tag{17}$$
$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & -\beta\\ \gamma & -\delta\end{array}\right]\right) \tag{18}$$
$$= \frac{\alpha-\delta}{2} \tag{19}$$

Thus, in more conventional notation

$$M = \frac{1}{2}\left[\left(\alpha+\delta\right)I + \left(\beta+\gamma\right)\sigma_x + i\left(\beta-\gamma\right)\sigma_y + \left(\alpha-\delta\right)\sigma_z\right] \tag{20}$$

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