GENERAL 2X2 MATRIX IN TERMS OF PAULI MATRICES

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.3.5.

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Any 2×2 matrix can be written as a linear combination of the three Pauli matrices and the unit matrix. That is, for an arbitrary matrix M we have

$$M=\sum_{\alpha}m_{\alpha}\sigma_{\alpha} \tag{1}$$

where the coefficients are found from

$$m_{\alpha}=\frac{1}{2}\mathrm{Tr}\left(M\sigma_{\alpha}\right) \tag{2}$$

We can write this out explicitly as follows

$$M=\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right] \tag{3}$$

We then get

$$
\begin{aligned}
m_0 &= \frac{1}{2}\mathrm{Tr}\left(M\sigma_0\right) && (4)\\
&= \frac{1}{2}\mathrm{Tr}\left(MI\right) && (5)\\
&= \frac{1}{2}\mathrm{Tr}\left(M\right) && (6)\\
&= \frac{\alpha+\delta}{2} && (7)\\
m_1 &= \frac{1}{2}\mathrm{Tr}\left(M\sigma_1\right) && (8)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}0 & 1\\ 1 & 0\end{array}\right]\right) && (9)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\beta & \alpha\\ \delta & \gamma\end{array}\right]\right) && (10)\\
&= \frac{\beta+\gamma}{2} && (11)\\
m_2 &= \frac{1}{2}\mathrm{Tr}\left(M\sigma_2\right) && (12)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}0 & -i\\ i & 0\end{array}\right]\right) && (13)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}i\beta & -i\alpha\\ i\delta & -i\gamma\end{array}\right]\right) && (14)\\
&= i\frac{\beta-\gamma}{2} && (15)\\
m_3 &= \frac{1}{2}\mathrm{Tr}\left(M\sigma_3\right) && (16)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right]\left[\begin{array}{cc}1 & 0\\ 0 & -1\end{array}\right]\right) && (17)\\
&= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc}\alpha & -\beta\\ \gamma & -\delta\end{array}\right]\right) && (18)\\
&= \frac{\alpha-\delta}{2} && (19)
\end{aligned}
$$

Thus, in more conventional notation

$$M = \frac{1}{2}\left[\left(\alpha+\delta\right)I + \left(\beta+\gamma\right)\sigma_x + i\left(\beta-\gamma\right)\sigma_y + \left(\alpha-\delta\right)\sigma_z\right] \qquad (20)$$

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