GENERAL 2X2 MATRIX IN TERMS OF PAULI MATRICES

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.3.5.

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Any 2×2 matrix can be written as a linear combination of the three Pauli matrices and the unit matrix. That is, for an arbitrary matrix M we have

$$M=\sum_{\alpha}m_{\alpha}\sigma_{\alpha} \tag{0.1}$$

where the coefficients are found from

$$m_{\alpha}=\frac{1}{2}\mathrm{Tr}\left(M\sigma_{\alpha}\right) \tag{0.2}$$

We can write this out explicitly as follows

$$M=\left[\begin{array}{cc}\alpha & \beta\\ \gamma & \delta\end{array}\right] \tag{0.3}$$

We then get

$$m_0 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_0\right) \tag{0.4}$$

$$= \frac{1}{2}\mathrm{Tr}\left(MI\right) \tag{0.5}$$

$$= \frac{1}{2}\mathrm{Tr}\left(M\right) \tag{0.6}$$

$$= \frac{\alpha+\delta}{2} \tag{0.7}$$

$$m_1 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_1\right) \tag{0.8}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array}\right]\left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right]\right) \tag{0.9}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} \beta & \alpha \\ \delta & \gamma \end{array}\right]\right) \tag{0.10}$$

$$= \frac{\beta+\gamma}{2} \tag{0.11}$$

$$m_2 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_2\right) \tag{0.12}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array}\right]\left[\begin{array}{cc} 0 & -i \\ i & 0 \end{array}\right]\right) \tag{0.13}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} i\beta & -i\alpha \\ i\delta & -i\gamma \end{array}\right]\right) \tag{0.14}$$

$$= i\frac{\beta-\gamma}{2} \tag{0.15}$$

$$m_3 = \frac{1}{2}\mathrm{Tr}\left(M\sigma_3\right) \tag{0.16}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} \alpha & \beta \\ \gamma & \delta \end{array}\right]\left[\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right]\right) \tag{0.17}$$

$$= \frac{1}{2}\mathrm{Tr}\left(\left[\begin{array}{cc} \alpha & -\beta \\ \gamma & -\delta \end{array}\right]\right) \tag{0.18}$$

$$= \frac{\alpha-\delta}{2} \tag{0.19}$$

Thus, in more conventional notation

$$M = \frac{1}{2}\left[\left(\alpha+\delta\right)I + \left(\beta+\gamma\right)\sigma_x + i\left(\beta-\gamma\right)\sigma_y + \left(\alpha-\delta\right)\sigma_z\right] \tag{0.20}$$

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