SPIN FLIP OF ELECTRON IN MAGNETIC FIELD

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.4.4.

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The fact that the rotation operator in 2-d spin space can be written in terms of the Pauli matrices as

$$U[R(\theta)] = e^{-i\theta\cdot\sigma/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}\left(\hat{\theta}\cdot\sigma\right) \tag{1}$$

allows us to do some calculations involving an electron in a magnetic field. We've seen that placing a magnetic moment that is due to angular momentum in a constant magnetic field causes the magnetic moment to precess about the direction of the field. The Hamiltonian of an electron with spin $\mathbf{S}$ in a constant field $\mathbf{B}$ is

$$\begin{aligned} H &= -\gamma\mathbf{S}\cdot\mathbf{B} && (2)\\ &= -\frac{\gamma\hbar}{2}\sigma\cdot\mathbf{B} && (3) \end{aligned}$$

where γ is the gyromagnetic ratio

$$\begin{aligned} \gamma &= \frac{-e}{m}\,(\text{SI}) = \frac{-e}{mc}\,(\text{CGS}) && (4)\\ &= 1.76\times10^{11}\ \text{s}^{-1}\text{T}^{-1} && (5)\\ &= 1.76\times10^{7}\ \text{s}^{-1}\text{G}^{-1} && (6) \end{aligned}$$

As an example, suppose we have an electron initially in the spin-up state, with $s_z = +\frac{\hbar}{2}$ and turn on a magnetic field of $\mathbf{B} = (100\text{ G})\,\hat{\mathbf{x}}$ at $t = 0$. As the applied field is perpendicular to the initial spin, the precession will cause the spin vector to rotate in the yz plane about the x axis. To find how long it takes the spin to flip, we need the propagator, which is

$$U(t) = e^{-iHt/\hbar} \tag{7}$$
$$= e^{i\gamma\sigma\cdot\mathbf{B}t/2} \tag{8}$$
$$= e^{i\gamma Bt\sigma_x/2} \tag{9}$$

Comparing with 1 we see that $U(t)$ is equivalent to a rotation operator with angle $\theta = -\gamma Bt$. Thus we have

$$U(t) = \cos\frac{\gamma Bt}{2}I + i\sin\frac{\gamma Bt}{2}\sigma_x \tag{10}$$
$$= \begin{bmatrix} \cos\frac{\gamma Bt}{2} & i\sin\frac{\gamma Bt}{2} \\ i\sin\frac{\gamma Bt}{2} & \cos\frac{\gamma Bt}{2} \end{bmatrix} \tag{11}$$

The electron's state as a function of time is thus

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle \tag{12}$$
$$= \begin{bmatrix} \cos\frac{\gamma Bt}{2} & i\sin\frac{\gamma Bt}{2} \\ i\sin\frac{\gamma Bt}{2} & \cos\frac{\gamma Bt}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{13}$$
$$= \begin{bmatrix} \cos\frac{\gamma Bt}{2} \\ i\sin\frac{\gamma Bt}{2} \end{bmatrix} \tag{14}$$

The state first has a 100% probability of being found with spin down when (using 6):

$$\sin\frac{\gamma Bt}{2} = 1 \tag{15}$$
$$t = \frac{\pi}{\gamma B} \tag{16}$$
$$= \frac{\pi}{(1.76\times 10^{7})(100)} \tag{17}$$
$$= 1.78\times 10^{-9}\text{ s} \tag{18}$$