

## SPIN FLIP OF ELECTRON IN MAGNETIC FIELD

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Chapter 14, Exercise 14.4.4.

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The fact that the rotation operator in 2-d spin space can be written in terms of the Pauli matrices as

$$U[R(\theta)] = e^{-i\theta\cdot\sigma/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}(\hat{\theta}\cdot\sigma) \quad (1)$$

allows us to do some calculations involving an electron in a magnetic field. We've seen that placing a magnetic moment that is due to angular momentum in a constant magnetic field causes the magnetic moment to precess about the direction of the field. The Hamiltonian of an electron with spin  $\mathbf{S}$  in a constant field  $\mathbf{B}$  is

$$H = -\gamma\mathbf{S}\cdot\mathbf{B} \quad (2)$$

$$= -\frac{\gamma\hbar}{2}\boldsymbol{\sigma}\cdot\mathbf{B} \quad (3)$$

where  $\gamma$  is the gyromagnetic ratio

$$\gamma = \frac{-e}{m} \text{ (SI)} = \frac{-e}{mc} \text{ (CGS)} \quad (4)$$

$$= 1.76 \times 10^{11} \text{ s}^{-1}\text{T}^{-1} \quad (5)$$

$$= 1.76 \times 10^7 \text{ s}^{-1}\text{G}^{-1} \quad (6)$$

As an example, suppose we have an electron initially in the spin-up state, with  $s_z = +\frac{\hbar}{2}$  and turn on a magnetic field of  $\mathbf{B} = (100 \text{ G})\hat{\mathbf{x}}$  at  $t = 0$ . As the applied field is perpendicular to the initial spin, the precession will cause the spin vector to rotate in the  $yz$  plane about the  $x$  axis. To find how long it takes the spin to flip, we need the propagator, which is

$$U(t) = e^{-iHt/\hbar} \quad (7)$$

$$= e^{i\gamma\sigma \cdot \mathbf{B}t/2} \quad (8)$$

$$= e^{i\gamma Bt\sigma_x/2} \quad (9)$$

Comparing with 1 we see that  $U(t)$  is equivalent to a rotation operator with angle  $\theta = -\gamma Bt$ . Thus we have

$$U(t) = \cos \frac{\gamma Bt}{2} I + i \sin \frac{\gamma Bt}{2} \sigma_x \quad (10)$$

$$= \begin{bmatrix} \cos \frac{\gamma Bt}{2} & i \sin \frac{\gamma Bt}{2} \\ i \sin \frac{\gamma Bt}{2} & \cos \frac{\gamma Bt}{2} \end{bmatrix} \quad (11)$$

The electron's state as a function of time is thus

$$|\psi(t)\rangle = U(t) |\psi(0)\rangle \quad (12)$$

$$= \begin{bmatrix} \cos \frac{\gamma Bt}{2} & i \sin \frac{\gamma Bt}{2} \\ i \sin \frac{\gamma Bt}{2} & \cos \frac{\gamma Bt}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (13)$$

$$= \begin{bmatrix} \cos \frac{\gamma Bt}{2} \\ i \sin \frac{\gamma Bt}{2} \end{bmatrix} \quad (14)$$

The state first has a 100% probability of being found with spin down when (using 6):

$$\sin \frac{\gamma Bt}{2} = 1 \quad (15)$$

$$t = \frac{\pi}{\gamma B} \quad (16)$$

$$= \frac{\pi}{(1.76 \times 10^7)(100)} \quad (17)$$

$$= 1.78 \times 10^{-9} \text{ s} \quad (18)$$