COUPLING OF PROTON'S MAGNETIC MOMENT TO EXTERNAL FIELD

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Exercise 14.5.1.

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We've dealt with the Zeeman effect in a lot of detail before, but Shankar deals with it using the approximation of negelecting the coupling of the proton's magnetic moment to the external magnetic field **B**. Using classical arguments, we can see why this is a reasonable approximation.

The proton, like the electron, has both orbital and spin angular momentum. The proton's spin is $\frac{\hbar}{2}$, the same as the electron, so its spin magnetic moment is given by

$$\mu_{ps} = \frac{q}{2Mc}\frac{\hbar}{2} \tag{1}$$

where M is the proton's mass. Since (apart from the sign) the proton and electron have the same charge q and spin, the equivalent formula for the electron is

$$\mu_{es} = \frac{q}{2mc}\frac{\hbar}{2} \tag{2}$$

where m is the electron mass. Thus

$$\mu_{ps} = \frac{m}{M}\mu_{es} \tag{3}$$

so that the proton's spin magnetic moment is about $\frac{1}{1836}$ times that of the electron.

For the orbital magnetic moment, we can consider a classical system in which the electron and proton are orbiting about their centre of mass. The period T of the orbit is the same for both particles, and the radius of each orbit is

$$r_p = \frac{m}{m+M}r \approx \frac{m}{M}r \tag{4}$$

$$r_e = \frac{M}{m+M}r \approx r \tag{5}$$

where r is the distance between the two particles. The orbital magnetic moment can be written as

$$\mu_i = \frac{qv_i r_i}{2c} \tag{6}$$

where the subscript i is either e or p. Since the proton moves in a smaller orbit but at the same frequency as the electron, its velocity is smaller. We have

$$v_p = \frac{2\pi r_p}{T} = \frac{2\pi r}{T}\frac{m}{m+M} \approx \frac{2\pi r}{T}\frac{m}{M} \tag{7}$$

$$v_e = \frac{2\pi r_e}{T} = \frac{2\pi r}{T}\frac{M}{m+M} \approx \frac{2\pi r}{T} \tag{8}$$

Therefore

$$\mu_p \approx \frac{\pi q r^2}{cT}\left(\frac{m}{M}\right)^2 \tag{9}$$

$$\mu_e \approx \frac{\pi q r^2}{cT} \tag{10}$$

Thus the orbital magnetic moment of the proton is about $\left(\frac{m}{M}\right)^2$ times that of the electron.

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