

## COUPLING OF PROTON'S MAGNETIC MOMENT TO EXTERNAL FIELD

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Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press.

Exercise 14.5.1.

[If some equations are too small to read easily, use your browser's magnifying option (Ctrl + on Chrome, probably something similar on other browsers).]

We've dealt with the Zeeman effect in a lot of detail before, but Shankar deals with it using the approximation of neglecting the coupling of the proton's magnetic moment to the external magnetic field  $\mathbf{B}$ . Using classical arguments, we can see why this is a reasonable approximation.

The proton, like the electron, has both orbital and spin angular momentum. The proton's spin is  $\frac{\hbar}{2}$ , the same as the electron, so its spin magnetic moment is given by

$$(1) \quad \mu_{ps} = \frac{q \hbar}{2Mc} \frac{1}{2}$$

where  $M$  is the proton's mass. Since (apart from the sign) the proton and electron have the same charge  $q$  and spin, the equivalent formula for the electron is

$$(2) \quad \mu_{es} = \frac{q \hbar}{2mc} \frac{1}{2}$$

where  $m$  is the electron mass. Thus

$$(3) \quad \mu_{ps} = \frac{m}{M} \mu_{es}$$

so that the proton's spin magnetic moment is about  $\frac{1}{1836}$  times that of the electron.

For the orbital magnetic moment, we can consider a classical system in which the electron and proton are orbiting about their centre of mass. The period  $T$  of the orbit is the same for both particles, and the radius of each orbit is

$$(4) \quad r_p = \frac{m}{m+M}r \approx \frac{m}{M}r$$

$$(5) \quad r_e = \frac{M}{m+M}r \approx r$$

where  $r$  is the distance between the two particles. The orbital magnetic moment can be written as

$$(6) \quad \mu_i = \frac{qv_i r_i}{2c}$$

where the subscript  $i$  is either  $e$  or  $p$ . Since the proton moves in a smaller orbit but at the same frequency as the electron, its velocity is smaller. We have

$$(7) \quad v_p = \frac{2\pi r_p}{T} = \frac{2\pi r}{T} \frac{m}{m+M} \approx \frac{2\pi r}{T} \frac{m}{M}$$

$$(8) \quad v_e = \frac{2\pi r_e}{T} = \frac{2\pi r}{T} \frac{M}{m+M} \approx \frac{2\pi r}{T}$$

Therefore

$$(9) \quad \mu_p \approx \frac{\pi q r^2}{cT} \left(\frac{m}{M}\right)^2$$

$$(10) \quad \mu_e \approx \frac{\pi q r^2}{cT}$$

Thus the orbital magnetic moment of the proton is about  $\left(\frac{m}{M}\right)^2$  times that of the electron.

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