EINSTEIN SOLID

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A simple model of a solid proposed by Einstein in 1907 is that it consists of a collection of N oscillators with quantized energy units. We can think of each oscillator as a quantum harmonic oscillator, and each energy unit as a quantum of size $\hbar\omega$, but the concept applies to any system with energy units that are all the same size. In general, a solid with N oscillators can have q energy units to distribute amongst them, so the number of possible microstates of such a system is the number of ways of distributing q balls into N bins. This is a standard problem in combinatorics, and the solution goes as follows.

We can represent the q balls by Xs and the N bins by $N-1$ vertical bars, where each bar serves to separate the contents of one bin from its neighbour. Thus if we have $N = 3$ bins and $q = 4$ balls, the possible microstates are

$$
\begin{gathered}
||XXXX \\
|XXXX| \\
XXXX|| \\
|X|XXX \\
|XX|XX \\
|XXX|X \\
X||XXX \\
XX||XX \\
XXX||X \\
X|XXX| \\
XX|XX| \\
XXX|X| \\
X|X|XX \\
X|XX|X \\
XX|X|X
\end{gathered}
$$

N	q	$\binom{q+N-1}{q}$
3	5	21
3	6	28
4	2	10
4	3	20
1	anything	1
anything	1	N
30	30	59132290782430712

TABLE 1. Number of microstates for N bins and q balls.

In general, the number of microstates is the number of ways of choosing q (or $N-1$) objects from a total of $q+N-1$ objects, without regard to order, which is just the binomial coefficient $\binom{q+N-1}{q}$. For the example just given,

$$\binom{q+N-1}{q} = \binom{6}{4} = 15 \tag{1}$$

which corresponds to the 15 cases listed above.

We could list the microstates for several other values of N and q, but this gets pretty tedious and the general idea should be obvious from the above. We'll just list the number of microstates for each case (see Table 1).

Well OK, just one more example of all the microstates, with $N = 4$ and $q = 2$.

$$\begin{array}{c} |||XX \\ ||XX| \\ |XX|| \\ XX||| \\ ||X|X \\ |X|X| \\ X|X|| \\ |X||X \\ X|||X \\ X||X| \end{array}$$

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