INTERNAL COMBUSTION ENGINE - THE OTTO CYCLE

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One example of a real heat engine is the internal combustion engine used in cars and machines that run on petrol (gasoline). This engine follows a cycle known as the Otto cycle, with steps as shown in the PV diagram in Fig. 1.

The working substance is a mixture of air and vaporized petrol contained in a cylinder with a piston at one end. Starting at point 1, the mixture is compressed adiabatically to point 2, at which point a spark from a spark plug ignites the mixture, causing a rapid increase in pressure at constant volume, taking us to point 3. From 3 to 4, the piston is pushed outwards causing an adiabatic expansion, which is where the engine does its work. At point 4, the piston is at its maximum extent and the gas is reduced in pressure at constant volume, releasing waste heat. [Actually, the step from

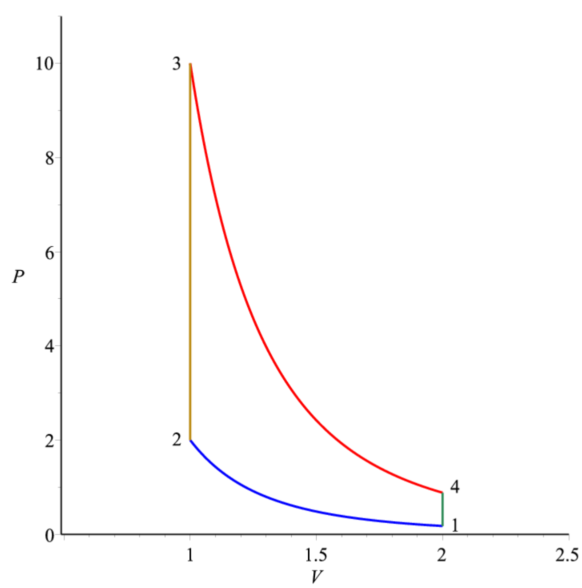

FIGURE 1. Otto cycle.

4 to 1 expels the gas through a valve and injects a fresh air/petrol mixture through another valve, in readiness for the next cycle.]

To draw an analogy with the heat engine, the Otto engine absorbs an amount of heat Q_h along path 2 to 3. This heat is generated not by contact with a hot reservoir, but by burning the fuel. Since stages 2 to 3 and 4 to 1 both occur at constant volume, no work is done there. The net work produced by the engine is that generated along the main power stroke from 3 to 4, minus the work required to compress the air/petrol mixture from 1 to 2. The efficiency is the net work divided by Q_h. We thus need to find both Q_h and W.

To calculate Q_h, we note that no work is done along path 2 to 3, so Q_h is equal to the increase in energy increase along this edge. The energy of a gas where each molecule has f degrees of freedom is given by the equipartition theorem as

$$U = \frac{f}{2}NkT \tag{1}$$

Combining this with the ideal gas law $PV = NkT$ we get

$$\begin{aligned} Q_h &= \frac{f}{2}Nk(T_3 - T_2) && (2) \\ &= \frac{f}{2}V_2(P_3 - P_2) && (3) \end{aligned}$$

[Note that the subscripts refer to the points in the above diagram, not to the actual values these quantities have. Thus V_2 is the volume at point 2, which in the above diagram actually has the value $V_2 = 1$. We're not interested in the actual values of the quantities in this derivation.]

Along an adiabatic curve, we have the relation

$$PV^{\gamma} = K \tag{4}$$

where $\gamma = (f+2)/f$ and K is a constant. We can therefore write Q_h as

$$Q_h = \frac{V_2(P_3 - P_2)}{\gamma - 1} \tag{5}$$

Along curve 3 to 4, we have

$$PV^{\gamma} = P_3 V_2^{\gamma} \tag{6}$$

while along curve 1 to 2:

$$PV^{\gamma} = P_2 V_2^{\gamma} \tag{7}$$

The net work produced along the two adiabatic curves is

$$W = \int_{3\to 4} P\,dV + \int_{1\to 2} P\,dV \tag{8}$$

$$= P_3 V_2^{\gamma} \int_{V_2}^{V_1} V^{-\gamma} dV + P_2 V_2^{\gamma} \int_{V_1}^{V_2} V^{-\gamma} dV \tag{9}$$

$$= -\frac{V_2^{\gamma}}{\gamma - 1}\left[P_3\left(V_1^{-(\gamma-1)} - V_2^{-(\gamma-1)}\right) + P_2\left(V_2^{-(\gamma-1)} - V_1^{-(\gamma-1)}\right)\right] \tag{10}$$

$$= \frac{V_2^{\gamma}}{\gamma - 1}(P_3 - P_2)\left(V_2^{-(\gamma-1)} - V_1^{-(\gamma-1)}\right) \tag{11}$$

$$= \frac{V_2}{\gamma - 1}(P_3 - P_2)\left[1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\right] \tag{12}$$

The efficiency is found by combining this result with 5

$$e = \frac{W}{Q_h} = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} \tag{13}$$

The ratio V_1/V_2 is known as the compression ratio (remember $V_1 > V_2$ so this ratio is always greater than 1. Thus $\left(\frac{V_2}{V_1}\right)^{\gamma-1} < 1$ and $0 < e < 1$).

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