

## QUASISTATIC ADIABATIC PROCESS IN AN IDEAL GAS

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We've seen that for an ideal gas undergoing a quasistatic (where at every stage the gas is infinitesimally close to equilibrium), adiabatic (no heat exchanged) process, the pressure  $P$  and volume  $V$  are related by

$$PV^\gamma = \text{constant} \quad (1)$$

where

$$\gamma \equiv \frac{2+f}{f} \quad (2)$$

and  $f$  is the number of degrees of freedom of each gas molecule.

This relation can also be derived by considering the heat capacities  $C_V$  at constant volume and  $C_P$  at constant pressure. We begin by considering the first law of thermodynamics which is just the conservation of energy. For an ideal gas, the change  $dE$  of energy due to adding an amount of heat  $dQ$  and compressing the gas by an amount  $dV$  is

$$dE = dQ - PdV \quad (3)$$

The minus sign for the  $-PdV$  term is because compressing the gas does work on the gas but the volume decreases so  $dV < 0$ . The d-bar symbol  $d$  is used to indicate that the corresponding quantity is not a state variable.

The heat capacities are defined as the heat change per unit temperature. At constant volume,  $dV = 0$  so the change in heat is due entirely to the addition of heat, and we have

$$C_V = \frac{dQ}{dT} = \left( \frac{\partial E}{\partial T} \right)_V \quad (4)$$

At constant pressure, the heat change is due to both added heat and a change in volume. From 3 we have

$$C_P = \frac{dQ}{dT} = \left( \frac{\partial E}{\partial T} \right)_P + P \left( \frac{\partial V}{\partial T} \right)_P \quad (5)$$

To proceed we use the experimental fact that, for an ideal gas, the internal energy depends only on the temperature  $T$ , so that

$$\left(\frac{\partial E}{\partial T}\right)_V = \left(\frac{\partial E}{\partial T}\right)_P \quad (6)$$

In an ideal gas, the molecules are essentially point particles that do not interact with each other.

Using this fact, we get from 4 and 5

$$C_P - C_V = P \left(\frac{\partial V}{\partial T}\right)_P \quad (7)$$

From the ideal gas law for a single mole of gas

$$PV = RT \quad (8)$$

we have

$$P \left(\frac{\partial V}{\partial T}\right)_P = P \frac{R}{P} = R \quad (9)$$

Thus the difference in heat capacities is the gas constant  $R$ :

$$C_P - C_V = R \quad (10)$$

For an adiabatic process  $\delta Q = 0$  since no heat is exchanged, so from 3

$$dE = -PdV \quad (11)$$

From 4 we have

$$dE = C_V dT \quad (12)$$

and from 8 and 10 we have

$$P = \frac{RT}{V} = (C_P - C_V) \frac{T}{V} \quad (13)$$

Substituting into 11 we have

$$C_V dT = - (C_P - C_V) \frac{T}{V} dV \quad (14)$$

Separating variables we have

$$\frac{dT}{T} = - \frac{C_P - C_V}{C_V} \frac{dV}{V} = -(\gamma - 1) \frac{dV}{V} \quad (15)$$

where  $\gamma$  is now defined as

$$\gamma \equiv \frac{C_P}{C_V} \quad (16)$$

Integrating we have

$$\ln T = -(\gamma - 1) \ln V + \ln K \quad (17)$$

where  $K$  is a constant of integration. Exponentiating both sides we have

$$T = KV^{1-\gamma} \quad (18)$$

From the ideal gas law 8 we have

$$T = \frac{PV}{R} \quad (19)$$

so

$$\frac{PV}{R} = KV^{1-\gamma} \quad (20)$$

Multiplying through by  $V^{\gamma-1}$  gives

$$PV^\gamma = RK = \text{constant} \quad (21)$$

Comparing with 2 we see that the ratio of heat capacities is related to the degrees of freedom on the molecules.

$$\frac{C_P}{C_V} = \frac{f+2}{f} \quad (22)$$