REFRIGERATORS IN THE REAL WORLD

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A real-life refrigerator follows a cycle that is essentially a backwards Rankine cycle. A PV diagram for this cycle is shown in Fig. 1.

The path $1 \to 2$ is an adiabatic compression of the refrigerant fluid (a common fluid is the hydrofluorocarbon HFC-134a) which is a gas along this path. At point 1, it is in contact with the cold reservoir (the inside of the fridge) and during compression (that's what the motor on your fridge is doing), its temperature rises to above that of the hot reservoir (the room), so at point 2, the refrigerant is a superheated gas (although 'superheated' in this case means only somewhat above the room temperature, not hundreds of degrees as in a steam engine). Along the edge $2 \to 3$, the gas condenses to a liquid while giving off heat Q_h to the surrounding room. On the back of a fridge, you'll usually find a radiator that serves to expel this heat.

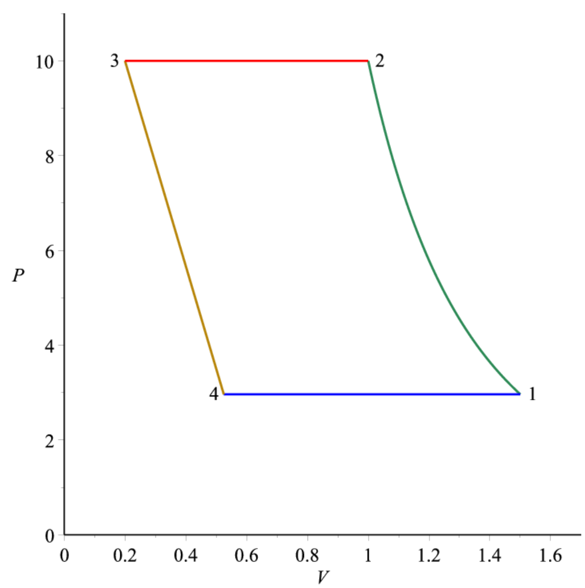

FIGURE 1. Backwards Rankine cycle.

The path $3 \to 4$ causes the fluid to reduce its temperature to a point lower than the inside of the fridge. Along $4 \to 1$, the fluid absorbs heat Q_c from the cold reservoir (the inside of the fridge) and in the process, turns from liquid into gas again.

The path $3 \to 4$ is the crucial step, and is known as *throttling*. It is assumed that this is adiabatic, so no heat is exchanged. The model used by Schroeder to explain throttling consists of a tube containing a porous plug in the middle. The fluid starts to the left of the plug at pressure P_3 in initial volume V_3 and is forced by a piston to pass through the plug into a chamber on the right, where the pressure is P_4 and the volume is V_4 after all the gas has passed through. Thus there is another piston on the right which moves outwards as the piston on the left moves inwards.

In terms of the phases of the fluid, point 1 is 100% saturated gas (that is, it's a gas but at the boiling point for that pressure), point 3 is 100% liquid (again, at the boiling point for that pressure). Point 2 is 'superheated' gas, and point 4 is part liquid, part gas.

Because it's an adiabatic process, there is no heat exchanged so the energy change of the fluid as it passes through the plug is due entirely to the difference in work done by the pistons on the two sides. On the left, the piston compresses the gas at constant pressure (that may sound impossible, but remember the gas is being forced through the plug as it is compressed, so really what we're doing is forcing a piston through a volume V_3 at constant pressure P_3), so the total work done is P_3V_3. On the right, the gas expands to fill volume V_4 against constant pressure P_4 exerted by the piston on the right, so the work done *on* the gas is negative and is $-P_4V_4$. The change in energy is therefore

$$U_4 - U_3 = P_3V_3 - P_4V_4 \tag{1}$$

In terms of enthalpy $H = U + PV$, so

$$H_4 = U_4 + P_4V_4 = U_3 + P_3V_3 = H_3 \tag{2}$$

That is, the enthalpy doesn't change in the throttling process.

The coefficient of performance (COP) of the refrigerator is the heat Q_c extracted divided by the work done, which is $Q_h - Q_c$. The heat transfer takes place entirely along the two edges where the pressure is constant, so the heat transfer is equal to the enthalpy change along each edge. That is

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_3 - (H_1 - H_4)} \tag{3}$$

$$= \frac{H_1 - H_4}{H_2 - H_1} \tag{4}$$

As with steam engines, the working fluid is not an ideal gas so we need to look up these enthalpies in tables. Schroeder provides a couple of tables (Tables 4.3 and 4.4) for HFC-234a which we can use for examples. The calculation procedure is very similar to that for steam engines.

Example 1. A refrigerator using HFC-234a operates between a high pressure of $P_3 = 12$ bars and a low pressure of $P_4 = 1$ bar. From Table 4.3, this makes $T_3 = 46.3°$ C and $T_4 = -26.4°$ C. Along the throttling path $3 \to 4$, since the enthalpy is constant, we can use the tables to find the fraction of liquid at point 4. We have $H_4 = H_3 = 116$ kJ (since at point 3, the fluid is 100% liquid). From the line for $P = 1$ bar in Table 4.3, we have

$$116 = 16x + 231(1-x) \tag{5}$$

$$x = 0.535 \tag{6}$$

Thus at point 4, the fluid is 53.5% liquid and 46.5% gas.

Example 2. A household fridge operates between $P_3 = 10$ bars and $P_4 = 1$ bar. Along the compression path $1 \to 2$, the fluid is 100% gas. From Table 4.3, the entropy at point 1 is $S_1 = 0.94$ kJ K^{-1}. Assuming the compression is adiabatic, the entropy doesn't change so, from Table 4.4, for $P_2 = 10$ bars we must have a temperature T_2 somewhere between 40 and 50, since the entropies for those two temperatures lie on either side of 0.94. That is

$$S_2 = S_1 = 0.94 = 0.907x + 0.943(1-x) \tag{7}$$

$$x = 0.0833 \tag{8}$$

$$T_2 = 40x + 50(1-x) = 49.17° \text{ C} \tag{9}$$

To find the enthalpies at all four points in the cycle, we can start by reading some of them from the tables. At point 1, $P_1 = 1$ bar and the fluid is 100% saturated gas, so from Table 4.1 we have

$$H_1 = 231 \text{ kJ} \tag{10}$$

At point 3, we have 100% saturated liquid at $P_3 = 10$ bars so

$$H_4 = H_3 = 105 \text{ kJ} \tag{11}$$

At point 2, we can take the same combination of enthalpies from Table 4.4 as we used to calculate the temperature:

$$H_2 = 269x + 280(1-x) = 279.08 \text{ kJ} \tag{12}$$

The COP is, from 4

$$\text{COP} = \frac{231 - 105}{279.08 - 231} = 2.62 \tag{13}$$

For a Carnot refrigerator, the maximum COP is

$$\text{COP}_{\text{max}} = \frac{T_c}{T_h - T_c} \tag{14}$$

The cold temperature T_c must be the warmest temperature on the edge $4 \rightarrow 1$, since heat must be able to flow from the fridge into the fluid all along this path. Since the entire path $4 \rightarrow 1$ falls within the region where the fluid is a saturated mixture of liquid and gas at the boiling point for pressure $P_1 = 1$ bar, the temperature here is constant at $T_c = -26.4^\circ$ C.

To find T_h, we apply the same logic to the path $2 \rightarrow 3$. Here, the fluid temperature must be above that of the room for the entire path so that heat can flow out into the room. However, part of the path lies in the superheated region, while part lies within the saturated liquid-gas region. It has its lowest temperature in this latter region. At point 3, the fluid is 100% liquid at $P_3 = 10$ bars so its temperature here is (from Table 4.3) 39.4° C so this is T_h. We have

$$\text{COP}_{\text{max}} = \frac{273 - 26.4}{39.4 + 26.4} = 3.75 \tag{15}$$

A temperature of $T_h = 39.4^\circ$C is reasonable, since it's unlikely that even in a hot kitchen the temperature would exceed this value (this is higher than normal body temperature). However, in very hot environments without air conditioning, it's certainly possible for the heat to reach this level, so such a fridge wouldn't work very well in those areas.

Finally, we can work out the liquid-gas proportion at point 4, by equating enthalpies $H_3 = H_4$ and interpolating as above:

$$H_3 = 105 = 16x + 231(1 - x) \tag{16}$$

$$x = 0.586 \tag{17}$$

Thus at point 4, the fluid is 58.6% liquid and 41.4% gas.

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