TWO-STATE PARAMAGNET AS AN EINSTEIN SOLID

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As a more-or-less real world example of an Einstein solid, we can look at a two-state paramagnet which is a material composed of magnetic dipoles which can align parallel or antiparallel to an external magnetic field. The restriction to two states comes from quantum mechanics, since the magnetic moment can take on only certain discrete values. The two-state paramagnet is the simplest case; in other magnets, the dipole can take on more than two states relative to the applied field.

We can model the two-state paramagnet as an Einstein solid where the role of the oscillators is played by the dipoles, and an energy quantum is the difference in energy between a parallel and antiparallel dipole. The antiparallel dipole has higher energy since a torque must be applied to twist the dipole against the field.

Using this model, we can picture two interacting paramagnets as two interacting Einstein solids. As before, we have two solids, A and B, containing N_A and N_B dipoles and q_A and q_B quanta of energy, with $q_A + q_B = q = $ constant. For any particular partition of the quanta, that is, for particular values of q_A and q_B, the total number of microstates available to the compound system is

$$\Omega_{\text{total}} = \Omega_A \Omega_B = \binom{q_A + N_A - 1}{q_A}\binom{q_B + N_B - 1}{q_B} \tag{1}$$

The overall number of microstates is

$$\Omega_{\text{overall}} = \binom{q + N_A + N_B - 1}{q} \tag{2}$$

Consider a system with $N_A = N_B = 100$ and $q = 80$. Using Maple to calculate the binomial coefficients (Maple has a 'binomial' function that does this automatically) and produce the plot, we have
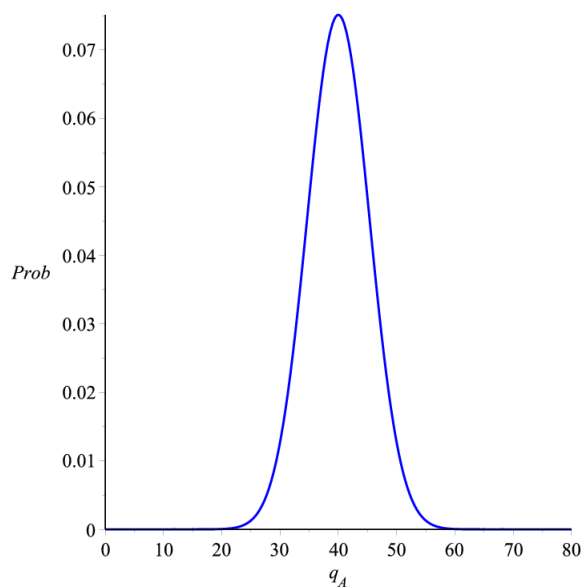
FIGURE 1. Probability distribution with $N_A = N_B = 100$ and $q = 80$.

$$\Omega_{\text{overall}} = \binom{80 + 100 + 100 - 1}{80} = 2.12 \times 10^{71} \tag{3}$$

$$\Omega_A = \binom{q_A + 99}{q_A} \tag{4}$$

$$\Omega_B = \binom{q - q_A + 99}{q - q_A} = \binom{179 - q_A}{80 - q_A} \tag{5}$$

$$Prob(q_A) = \frac{\Omega_A \Omega_B}{\Omega_{\text{overall}}} \tag{6}$$

A plot of the probabilities is in Fig. 1.

As we'd expect, the most probable state is $q_A = q_B = 40$, with a probability of 0.075. The least probable state is either $q_A = 0$ or $q_A = 80$, both of which have probability 7.87×10^{-20}.

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