GEOMETRIZED UNITS

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References: Kip S. Thorne & Roger D. Blandford, *Modern Classical Physics*, Princeton University Press (2017). Exercises 1.15, 2.1.

Post date: 5 Oct 2020.

In SI units:

$$\begin{aligned} G &= 6.67\times10^{-11}\ \mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2} \\ c &= 3.00\times10^{8}\ \mathrm{m}\cdot\mathrm{s}^{-1} \\ \hbar &= 1.05\times10^{-34}\ \mathrm{m}^2\mathrm{kg}\cdot\mathrm{s}^{-1} \\ m_e &= 9.11\times10^{-31}\ \mathrm{kg} \end{aligned} \tag{1}$$

We are asked to convert several expressions from the geometrized form in which $c=1$. Note that this isn't the same as the 'natural units' that are used in quantum theory, where $\hbar=1$ as well. In general relativity, G is also usually set to 1, but we won't do that yet.

Ex 1.15(a) For the Planck time, we are given

$$t_P=\sqrt{G\hbar} \tag{2}$$

From 1, the units of $G\hbar$ are $\mathrm{m}^5\mathrm{s}^{-3}$. To get t_P in seconds, we need to insert a power of c such that the units of $G\hbar$ come out to seconds squared. Since c has units $\mathrm{m}\cdot\mathrm{s}^{-1}$, we see that dividing by c^5 will do this. Thus we have

$$t_P=\sqrt{\frac{G\hbar}{c^5}}=5.37\times10^{-44}\ \mathrm{s} \tag{3}$$

To get this in metres, we multiply by c:

$$t_P=1.61\times10^{-35}\ \mathrm{m} \tag{4}$$

A time expressed as a distance is the distance light travels in that time.

Ex 1.15(b) The energy $\mathcal{E}=2m_e$ obtained from the annihilation of an electron and positron can be converted to SI units by multiplying by c^2:

$$\mathcal{E}=2m_ec^2=1.64\times10^{-13}\ \text{joules} \tag{5}$$

Ex. 1.15(c) I'm not sure what we're expected to do here, since the Lorentz force law given in the question is already in SI units:

$$\boldsymbol{F}=m\boldsymbol{a}=m\frac{d\boldsymbol{v}}{dt}=q\left(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}\right) \tag{6}$$

We can write the equation in CGS units as

$$\boldsymbol{F}=q_{cgs}\left(\boldsymbol{E}_{cgs}+\frac{\boldsymbol{v}}{c}\times\boldsymbol{B}_{cgs}\right) \tag{7}$$

where the subscript cgs indicates that these quantities are expresed in CGS units.

Ex. 1.15(d) The geometric form of the momentum of a photon is

$$\boldsymbol{p}=\hbar\omega\boldsymbol{n} \tag{8}$$

where ω is the angular frequency and $\boldsymbol{n}$ is a unit vector in the direction of propagation. In SI units 1 the units of $\hbar\omega$ are $\text{m}^2\text{kg}\cdot\text{s}^{-1}\cdot\text{s}^{-1}=\text{m}^2\text{kg}\cdot\text{s}^{-2}$, and the units of momentum are $\text{m}\cdot\text{kg}\cdot\text{s}^{-1}$, so to convert to SI units, we need to divide 8 by c:

$$\boldsymbol{p}=\frac{\hbar\omega}{c}\boldsymbol{n} \tag{9}$$

Ex. 1.15(e) My height is 181 cm = 1.81 m, so my height in seconds is

$$H=\frac{1.81}{c}=6.03\times10^{-9}\text{ s} \tag{10}$$

My age is (at the time of writing) almost 67 years, so in metres we have

$$A=\left(67\text{ years}\times365.25\text{ days}\cdot\text{year}^{-1}\times86400\text{ s}\cdot\text{day}^{-1}\right)c \tag{11}$$

$$=6.34\times10^{17}\text{ m} \tag{12}$$

which is, of course, 67 light years.

A quick Google search tells us that there around 600 to 700 stars within 60 light years of the Sun, so if we were able to travel at the speed of light, we'd be able to visit a fair number in my lifetime.

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