NUMERICS OF COMPONENT MANIPULATIONS

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References: Kip S. Thorne & Roger D. Blandford, *Modern Classical Physics*, Princeton University Press (2017). Exercise 2.6.

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Here is a numerical example of using tensor components in 4-d spacetime. We are given a vector whose only non-zero components are

$$\begin{aligned} A^0 &= 1 \\ A^1 &= 2 \end{aligned} \tag{1}$$

and a rank-2 tensor T whose only non-zero components are

$$\begin{aligned} T^{00} &= 3 \\ T^{10} &= T^{01} = 2 \\ T^{11} &= -1 \end{aligned} \tag{2}$$

We have

$$\mathsf{T}\left(\vec{A},\vec{A}\right) = T^{\alpha\beta}A_\alpha A_\beta \tag{3}$$

Remember that raising or lowering a 0 index changes the sign, so we have

$$\begin{aligned} A_0 &= -1 \\ A_1 &= 2 \end{aligned} \tag{4}$$

and

$$T^{\alpha\beta}A_\alpha A_\beta = T^{00}A_0A_0 + T^{10}A_1A_0 + T^{01}A_0A_1 + T^{11}A_1A_1 \tag{5}$$

$$= 3-4-4-4 \tag{6}$$

$$= -9 \tag{7}$$

If we insert a vector into T's first slot and leave the second slot vacant, we get a vector $\mathsf{T}\left(\vec{A},\_\right)$. The components are

$$\mathsf{T}\left(\vec{A},\_\right)^{\alpha}=T^{\gamma\alpha}A_{\gamma} \tag{8}$$

so we have

$$\begin{aligned} T^{\gamma 0}A_{\gamma} &= T^{00}A_{0}+T^{10}A_{1} && (9)\\ &= -3+4 && (10)\\ &= 1 && (11)\\ T^{\gamma 1}A_{\gamma} &= T^{01}A_{0}+T^{11}A_{1} && (12)\\ &= -2-2 && (13)\\ &= -4 && (14) \end{aligned}$$

Finally, we have

$$\left(\vec{A}\otimes\mathsf{T}\right)^{\alpha\beta\gamma}=A^{\alpha}T^{\beta\gamma} \tag{15}$$

In this case, there is no sum over indices and we want the contravariant components so we use the contravariant components of $\vec{A}$ in 1. The non-zero components of the tensor product are

$$\begin{aligned} A^{0}T^{00} &= 3\\ A^{0}T^{01} &= A^{0}T^{10}=2\\ A^{0}T^{11} &= -1\\ A^{1}T^{00} &= 6\\ A^{1}T^{01} &= A^{1}T^{10}=4\\ A^{1}T^{11} &= -2 \end{aligned} \tag{16}$$