SIMULTANEITY IN SPECIAL RELATIVITY

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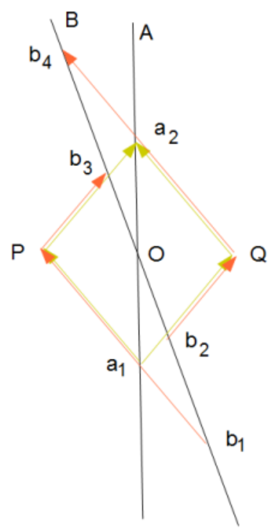
Reference: d'Inverno, Ray, *Introducing Einstein's Relativity* (1992), Oxford Uni Press. - Sections 2.10 and Problem 2.6.

One of the less intuitive predictions of special relativity is that two events which appear to be simultaneous to one observer will not necessarily appear simultaneous to another inertial observer moving relative to the first one. One of the simpler ways of seeing how this comes about is by examining a space-time diagram as shown:



The diagram is shown from the point of view of observer A, who sees the second observer B moving with a speed v to the left. The two observers meet at event O, at which point their clocks are synchronized.

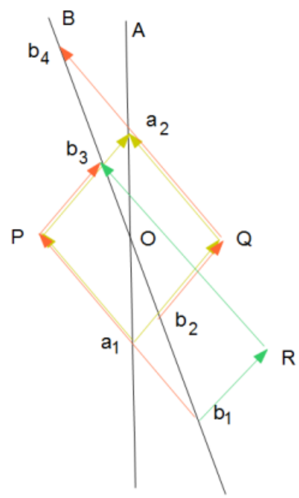
Both observers view two events, P and Q, by reflecting light beams off them. These two events are seen as simultaneous to A, since he emits two light beams at the same time a_1 (yellow arrows). These light beams bounce off P and Q and arrive back at A at the same time a_2. Thus, using our measures of time and distance from the last post, A says that both P and Q are at a distance $\frac{1}{2}(a_2-a_1)$ and both events occurred at time $\frac{1}{2}(a_2+a_1)$.

For B to detect event P, he has to send out a light beam at time (in B's frame) b_1 (orange arrow). This beam is reflected by P and arrives back at B at time b_3, so B says the event occurred at $(x_P,t_P)=\left(-\frac{1}{2}(b_3-b_1),\frac{1}{2}(b_3+b_1)\right)$.

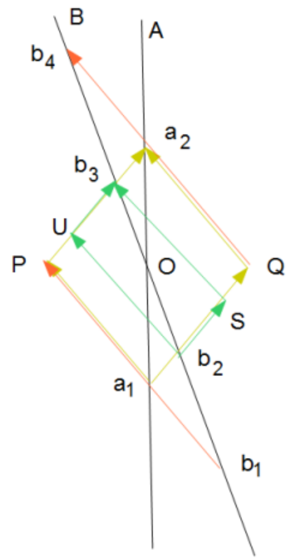
(The minus sign for x_P arises because the light beam is emitted to the left.) Similarly, event Q is detected by emitting a light beam at time b_2 and detecting its reflection at time b_4, so its coordinates appear to B as $(x_Q, t_Q) = \left(\frac{1}{2}(b_4 - b_2), \frac{1}{2}(b_4 + b_2)\right)$.

From the symmetry of the diagram, the time intervals $b_3 - b_1$ and $b_4 - b_2$ are the same, so A and B would agree that the two events P and Q occurred at the same distance on either side of O. However, it's quite clear from the diagram that the midpoint of the time interval b_1 to b_3 (that is, B's measurement of the time of P) is located *before* O and the midpoint of the time interval b_2 to b_4 (that is, B's measurement of the time of Q) is located *after* O, so in B's frame, event P happens first, then A and B meet at event O, then event Q happens.

If we wanted an event R that appeared to B to be simultaneous with event P, we would need an event that could be detected by firing off a light beam at time b_1 and receiving a reflection back at time b_3. Such an event R is shown in this diagram, where the new light beams are drawn in green:

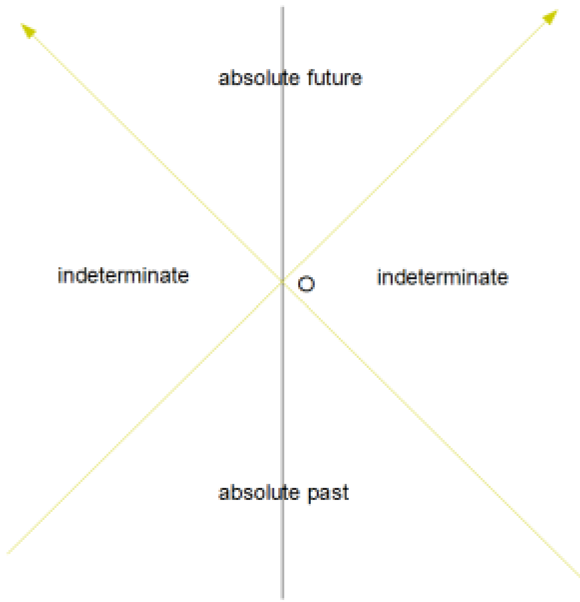


If we wanted an event S that appeared to B to be simultaneous with O, we would need an event that B could detect by firing off a light beam at a time $O - t$, say, and receiving a reflection back at time $O + t$. That is, the sending and receiving of the light beam must occur at equal times on either side of O. Since the times b_2 and b_3 in our diagram above satisfy that condition, we could arrange for event S to be one that receives a light beam from b_2 and reflects it back so that it arrives at B at time b_3 as shown:

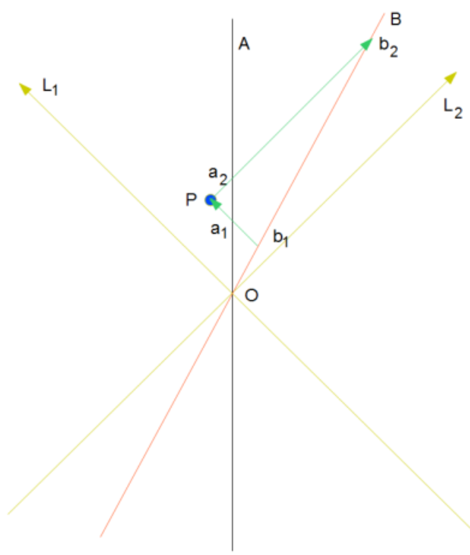


We've added another event U which is also simultaneous with O as seen by B. In fact, any event that lies on the straight line UOS appears to be simultaneous with O as seen by B.

We can use these diagrams to divide up space-time into four regions. If we draw in the light world lines from a given event O, as in the diagram, then any event in the top quadrant is in the absolute future, in the sense that all inertial systems will agree that events in this quadrant occur after O. Similarly, events in the bottom quadrant are in the absolute past, and all observers will agree that they occur before O. Events in the left and right quadrants are indeterminate, in that different observers will disagree about the ordering of these events. Events on the light world lines themselves are also determinate.



To see this, suppose we have an event P (blue dot in following diagram) inside the top quadrant.

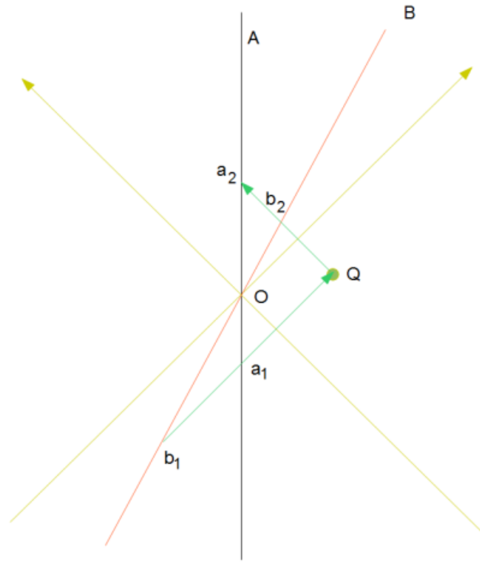


Two observers as shown will observe the event by sending light signals that depart at a_1 and b_1 respectively, and arrive at a_2 and b_2. Because all world lines of observers must lie inside the top quadrant (lines that lie inside the left or right quadrants would represent observers travelling faster than light, which isn't allowed), any light signal from any observer that is to intercept the event has to leave the observer on a world line that is parallel to and above one of the two light lines L_1 and L_2 that define the top quadrant.

Any observer who wishes to detect event O would need to emit signals that lie along lines L_1 or L_2 and receive a response along L_2 or L_1 respectively. These signals always lie below those used to detect event P, so O will always be seen as previous to P.

It's important to note that the two observers will disagree about the actual time at which P occurs, but they will agree that P happens after O.

In the case of an event Q in the right quadrant (see figure below), we can find two observers who disagree about the order in which Q and O occur.



In the diagram, A says that Q occurs at time $\frac{1}{2}(a_1 + a_2)$, which is *after* O (since the distance from O to a_1 in the diagram is clearly less than the distance from O to a_2). However, B says that Q occurs at time $\frac{1}{2}(b_1 + b_2)$, which clearly occurs before O in B's frame.

If we choose an observer whose world line lies between A and B at just the right angle, we would find that observer saying that O and Q were simultaneous.