SOLAR SAILS

We’ve seen that because electromagnetic waves carry momentum, they exert pressure on any surface they strike. The radiation pressure from the Sun at the distance of the Earth is

\[ P = 4.33 \times 10^{-6} \text{ N m}^{-2} \]  \hspace{1cm} (1)

This is for a perfect absorber. For a perfect reflector, the pressure is double this, since the light’s momentum is reversed rather than just stopped.

Suppose we wanted to accelerate a satellite radially away from the Sun starting at the Earth’s orbit, with an initial acceleration of \(1 \text{ g} = 9.8 \text{ m s}^{-2}\). If we want the force required to give this acceleration to be provided by radiation pressure from the Sun on a circular solar sail, how large does the sail have to be?

First, we should check whether we need to include the Sun’s gravity in the calculation (we’re assuming that the satellite is far enough from Earth that Earth’s gravity is negligible). At the distance of Earth, the acceleration due to the Sun’s gravity is

\[ a = \frac{GM_S}{r^2} \]  \hspace{1cm} (2)

\[ = \frac{(6.67384 \times 10^{-11})(1.989 \times 10^{30})}{(1.496 \times 10^{11})^2} \]  \hspace{1cm} (3)

\[ = 5.93 \times 10^{-3} \text{ m s}^{-2} \]  \hspace{1cm} (4)

Since our initial acceleration away from the Sun is \(9.8 \text{ m s}^{-2}\), the Sun’s gravity is also negligible at the distance of the Earth’s orbit.

If the sail is a perfect absorber, then if the satellite has mass \(m\), the area \(A\) of the sail must be
\begin{align*}
ma &= PA \quad \text{(5)} \\
A &= \frac{9.8m}{4.33 \times 10^{-6}} \quad \text{(6)}
\end{align*}

If \( m = 1.2 \times 10^4 \) kg then

\[ A = 2.716 \times 10^{10} \text{ m}^2 \quad \text{(7)} \]

giving a radius of

\[ r = \sqrt{\frac{A}{\pi}} = 93 \text{ km} \quad \text{(8)} \]

If the sail is a perfect reflector, the pressure is doubled, so the required area is \( A/2 \) and the radius is \( r/\sqrt{2} = 66 \) km. In practice, an acceleration of \( 1g \) is much greater than what can be expected from solar sails used up to date. See the Wikipedia article for more details.