RELATIVISTIC ACCELERATION IN TERMS OF FORCE

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Ordinary force in relativity is given by

\[ F = \frac{m}{\sqrt{1 - u^2/c^2}} \left[ a + \frac{(u \cdot a) u}{c^2 - u^2} \right] \]  

(1)

To get a general expression for the acceleration \( a \) in terms of the force, we take the dot product of both sides with the velocity \( u \):

\[ u \cdot F = \gamma m (u \cdot a) \left( 1 + \frac{u^2}{c^2 - u^2} \right) \]

(2)

\[ = \frac{\gamma mc^2 (u \cdot a)}{c^2 - u^2} \]

(3)

\[ = \gamma^3 m (u \cdot a) \]

(4)

\[ u \cdot a = \frac{u \cdot F}{\gamma^3 m} \]

(5)

Substituting back into (1) we get

\[ a = \frac{F}{\gamma m} - \frac{u}{c^2 - u^2} \frac{u \cdot F}{\gamma^3 m} \]

(6)

\[ = \frac{F}{\gamma m} - \frac{u}{\gamma mc^2} (u \cdot F) \]

(7)

In the limit of small \( u \), this reduces to the familiar Newton’s law \( F = ma \), but in the relativistic region, the acceleration depends on the object’s velocity. As a result, the acceleration isn’t parallel to the force unless \( F \) is either parallel to \( u \) or \( F \perp u \); in the latter case \( u \cdot F = 0 \) and the second term is zero.

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