COMPTON EFFECT; COMPTON WAVELENGTH

Link to: physicspages home page.
To leave a comment or report an error, please use the auxiliary blog.
Post date: 1 Aug 2015.
A while back, we derived the formula for the Compton effect, which is the change in the frequency of a photon when it scatters off a stationary electron. The formula, in relativistic units \((c = 1)\) is

\[
\frac{1}{\nu'} = \frac{1}{\nu} + \frac{\hbar}{m}(1 - \cos \theta) \tag{1}
\]

To convert this formula to wavelengths and to restore \(c\), we first reinsert \(c\) into the above equation. The units of \(\nu\) are \(s^{-1}\), and of \(m\) are \(kg^{-1}\). The units of Planck’s constant can be found from its definition in the formula

\[
E = h\nu \tag{2}
\]

Thus the units of \(h\) are \((\text{energy}) \times (\text{time}) = \text{kg m}^2\text{s}^{-1}\). This also happens to be the units of angular momentum, since \(L = r \times p\), so the units of \(L\) are \(m \times \text{kg m s}^{-1} = \text{kg m}^2\text{s}^{-1}\). From \(L\) the units of the last term on the RHS must be seconds, so with \(c\) present explicitly, we must have

\[
\frac{1}{\nu'} = \frac{1}{\nu} + \frac{\hbar}{mc^2}(1 - \cos \theta) \tag{3}
\]

In terms of wavelength, we have

\[
\lambda\nu = c \tag{4}
\]

so

\[
\frac{\lambda'}{c} = \frac{\lambda}{c} + \frac{\hbar}{mc^2}(1 - \cos \theta) \tag{5}
\]

\[
\Delta \lambda = \frac{\hbar}{mc}(1 - \cos \theta) \tag{6}
\]

The change in wavelength is inversely proportional to the mass of the stationary object off which the photon scatters. For an electron
\[ \frac{h}{m_e c} = \frac{6.62606957 \times 10^{-34}}{(9.10938291 \times 10^{-31})(2.99792458 \times 10^8)} = 2.426 \times 10^{-12} \text{ m} \] (7)

This is the Compton wavelength of the electron. The wavelength shift is independent of the wavelength of the incoming photon, so for visible light, where \( \lambda \) is around \( 5 \times 10^{-7} \) m, the Compton effect is negligible. It becomes noticeable only for much shorter wavelengths (and thus much higher energy photons). For X-rays and gamma rays, \( \lambda \) is in the range \( 10^{-12} - 10^{-11} \) m so the Compton shift is comparable with the original wavelength.

For a heavier particle, such as a proton, the Compton effect is correspondingly smaller. The Compton wavelength of a proton is

\[ \frac{h}{m_p c} = \frac{6.62606957 \times 10^{-34}}{(1.67262178 \times 10^{-27})(2.99792458 \times 10^8)} = 1.321 \times 10^{-15} \text{ m} \] (8)