To see that gravity is much weaker than the electrostatic force, we can repeat Bohr’s semi-classical derivation of the hydrogen energy levels, replacing the Coulomb force with the Newtonian gravitational force. We can do this with the following replacement:

\[
\frac{e^2}{4\pi \epsilon_0} \rightarrow Gm_e m_p
\]  

(1)

giving energy levels of

\[
E = - (Gm_e m_p)^2 \frac{m_e}{2n^2 \hbar^2}
\]  

(2)

\[
= - \frac{4.23 \times 10^{-97}}{n^2} \text{ J}
\]  

(3)

\[
= - \frac{2.64 \times 10^{-78}}{n^2} \text{ eV}
\]  

(4)

compared to the actual energy levels of hydrogen:

\[
E = - \frac{13.6}{n^2} \text{ eV}
\]  

(5)

The radii are

\[
r_n = \frac{n^2 \hbar^2}{Gm_e^2 m_p}
\]  

(6)

\[
= (1.2 \times 10^{29} \text{ m}) n^2
\]  

(7)

\[
= (1.2 \times 10^{38} \text{ nm}) n^2
\]  

(8)

\[
= (8 \times 10^{17} \text{ AU}) n^2
\]  

(9)

\[
= (1.27 \times 10^{13} \text{ ly}) n^2
\]  

(10)
Thus the ground state radius of gravitational hydrogen is many times larger than the visible universe, compared with the electrostatic Bohr radius of $5.29177 \times 10^{-11}$ m.