TRANSFORMATIONS OF SCALAR AND VECTOR FIELDS

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Reference: Sidney Coleman, *Quantum Field Theory: Lectures of Sidney Coleman* (edited by Bryan Gin-ge Chen *et al.*), World Scientific, 2019. Section 3.2.

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We summarize here some references for the results given in Section 3.2 on transformations of fields.

For the case of a rotation given by a rotation matrix R, a scalar field transforms according to

$$\phi'\left(\boldsymbol{x}'\right)=\phi\left(R^{-1}\boldsymbol{x}\right) \tag{1}$$

where

$$\boldsymbol{x}=\left(x^{1},x^{2},x^{3}\right) \tag{2}$$

is the position vector.

For a vector field $\boldsymbol{V}$, the transformation is

$$\boldsymbol{V}'\left(\boldsymbol{x}'\right)=R\boldsymbol{V}\left(R^{-1}\boldsymbol{x}\right) \tag{3}$$

That is, we need to apply the *inverse* rotation to the coordinates and the *direct* rotation to the vector itself.

A similar argument applies to other transformations such as a Lorentz transformation given by Λ. The scalar field transforms as

$$\phi'\left(\boldsymbol{x}'\right)=\phi\left(\Lambda^{-1}\boldsymbol{x}\right) \tag{4}$$

and a vector field transforms as

$$\boldsymbol{V}'\left(\boldsymbol{x}'\right)=\Lambda\boldsymbol{V}\left(\Lambda^{-1}\boldsymbol{x}\right) \tag{5}$$

Coleman gives the general rule as

$$\phi^{a}\left(x\right)\xrightarrow{\Lambda}\phi^{a\prime}\left(x'\right)=S_{b}^{a}\phi^{b}\left(\Lambda^{-1}x\right) \tag{6}$$

Here, the object S_{b}^{a} is one factor of Λ for each tensor index in the field ϕ^{b}. For a scalar field, there are no tensor indices so $S_{b}^{a}=1$ and we get 4. For a vector field, b runs over the values 1,2,3 (one for each vector component), so in matrix notation S_{b}^{a} is the (a,b) element of the matrix Λ. Written out as a matrix equation, we get 5.