CLASSICAL PARTICLE MECHANICS

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Reference: Sidney Coleman, *Quantum Field Theory: Lectures of Sidney Coleman* (edited by Bryan Gin-ge Chen *et al.*), World Scientific, 2019. Section 4.1.

Post date: 20 Dec 2019.

Chapter 4 gives an alternative route to quantum field theory by considering four 'boxes'. The first box is classical particle mechanics, the second is quantum particle mechanics (typically covered in a course on non-relativistic quantum theory), the third is classical field theory and the so-called 'missing box' is quantum field theory.

We've already looked at classical particle mechanics, so I'll just give a summary here.

Classical particle mechanics is obtained by specifying a Lagrangian $L(q^a,\dot{q}^a,t)$ as a function of generalized coordinates q^a, their time derivatives and the time t. We then define the action S as the integral of the Lagrangian between two times t_1 and t_2 and minimize the action between these times. More details are given in my earlier post. The result is the Euler-Lagrange equations

$$\frac{\partial L}{\partial q^a}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^a}\right)=0 \tag{1}$$

In Coleman's notation, he defines the canonical momentum p_a conjugate to q_a by

$$p_a\equiv\frac{\partial L}{\partial \dot{q}^a} \tag{2}$$

This gives the equations of motion

$$\frac{\partial L}{\partial q^a}-\frac{dp_a}{dt}=0 \tag{3}$$

An alternative to the Lagrangian formulation is the Hamiltonian formulation, which is obtained by doing a Legendre transformation. We introduce the Hamiltonian H as

$$H\equiv p_a\dot{q}^a-L \tag{4}$$

where H is now considered to be a function of the q^as and p_as, rather than q^as and $\dot{q}^a$s. By varying the Hamiltonian and assuming that the q^as and p_as are complete and independent variables, we arrive at Hamilton's equations

$$\frac{\partial H}{\partial p_a} = \dot{q}^a \tag{5}$$

$$\frac{\partial H}{\partial q^a} = -\dot{p}_a \tag{6}$$

Coleman points out that in order for Hamilton's equations to be valid, the condition of completeness and independence of the q^as and p_as must hold. Completeness means that it is possible to write H as a function of the q^as and p_as and no other variables. Independence means that it is possible to vary each of the q^as and p_as one at a time without affecting any of the others. Coleman points out that the independence condition can be violated if we introduce some constraints into the physical system. He gives the example of a particle constrained to move on the surface of a sphere. Constraints can be introduced into the Lagrangian by means of Lagrange multipliers. A Lagrange multiplier λ plays the role of another generalized coordinate, but its time derivative never appears in a Lagrangian, so its conjugate momentum is always zero, with the consequence that it cannot be varied. The derivation of Hamilton's equations breaks down in this case.