PARITY

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Reference: Sidney Coleman, *Quantum Field Theory: Lectures of Sidney Coleman* (edited by Bryan Gin-ge Chen *et al.*), World Scientific, 2019. Section 6.3.

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We've looked at the parity operator in some detail earlier, so I'll just summarize a few points made by Coleman in this post. The main effect of parity is to reverse all 3 spatial coordinates but not the time coordinate, so that $\boldsymbol{x}\to-\boldsymbol{x}$ and $t\to t$. Ordinary scalars such as mass are invariant under parity. However, there are some vectors which reverse under parity and some which don't, and there are also some scalars that change sign. Since velocity is the derivative of position (which changes sign) with respect to time (which doesn't), velocity changes sign so that $\boldsymbol{v}\to-\boldsymbol{v}$. Momentum is the product of an ordinary scalar m with $\boldsymbol{v}$ so it also changes sign: $\boldsymbol{p}\to-\boldsymbol{p}$. A vector which is a cross product of two ordinary vectors (vectors that do change sign under parity) picks up two minus signs which cancel out, so such a cross product does *not* change sign under parity. An example is angular momentum $\boldsymbol{L}=\boldsymbol{r}\times\boldsymbol{v}$. Vectors that do *not* change under parity are called *axial vectors*. (Vectors that *do* change sign under parity don't have any special name as far as I can see; they are just 'ordinary' vectors.)

A scalar formed by a triple product of 3 ordinary vectors such as $w=\boldsymbol{a}\cdot(\boldsymbol{b}\times\boldsymbol{c})$ picks up 3 minus signs so it *does* change sign. Such as scalar is called a *pseudoscalar*.

The behaviour of a collection of fields under parity can vary, depending on the properties of the fields. We could have fields of any of the four types (scalar, pseudoscalar, vector and axial vector), and we can also have field theories consisting of collections of more than one field, where the types of the constituent fields can be different. In such cases, there is no simple rule for determining the behaviour of the collection of fields under parity. What Coleman does assume, however, is that the action of parity on such a collection of fields is at worst a linear combination of the fields, so we have

$$P:\;\;\phi^{a}(\boldsymbol{x},t)\to M_{b}^{a}\phi^{b}(-\boldsymbol{x},t) \tag{1}$$

where the a,b indexes indicate which field we're considering, and M_b^a is some matrix of coefficients used to combine the fields under a parity transformation.

Coleman then gives a few examples of Lagrangians that behave in various ways under parity. The simplest example is for a single scalar field $\phi(\boldsymbol{x},t)$ with Lagrangian

$$\mathscr{L}^{(1)} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 - g\phi^4 \tag{2}$$

The superscript (1) just indicates that it is the Lagrangian in the first example.

For a single field, 1 states that

$$\phi(\boldsymbol{x},t) \to M\phi(-\boldsymbol{x},t) \tag{3}$$

for some constant M. If we take $M = 1$, then the Lagrangian transforms as

$$\mathscr{L}^{(1)}(\boldsymbol{x},t) \to \mathscr{L}^{(1)}(-\boldsymbol{x},t) \tag{4}$$

since the field appears in $\mathscr{L}^{(1)}$ raised to only even powers in 2. Since the action is the integral of $\mathscr{L}^{(1)}$ over all space, the substitution $\boldsymbol{x} \to -\boldsymbol{x}$ has no effect on the action and this system is invariant under parity.

If we take $M = -1$ in 3, the Lagrangian transforms the same way as in 4, so the system is invariant under this choice for parity as well.

Coleman gives 3 other examples of increasing complexity, not all of which correspond to actual physical systems. In some of these examples, it is impossible to find a definition of parity that leaves the system invariant, which in other cases, some definitions work while others don't.

In real life, most physical systems are parity-invariant, but in 1957 it was discovered that the weak interaction (involved in beta decay) actually does not conserve parity.