In an earlier post, we saw that placing a neutral atom in an external field induces a dipole moment \( p \) where for small fields the experimentally determined relation is

\[
p = \alpha E
\]  

(1)

where \( \alpha \) is the atomic polarizability.

If this external field is due to a point charge \( q \) at a distance \( r \) from the atom then the field at the atom due to the charge is

\[
E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2}
\]  

(2)

The induced dipole moment is therefore

\[
p = \frac{\alpha}{4\pi \epsilon_0} \frac{q}{r^2}
\]  

(3)

We’ve seen that the electric field due to a dipole is (in spherical coordinates):

\[
E = \frac{p}{4\pi \epsilon_0 r^3} \left[ 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right]
\]  

(4)

The field from the dipole induced by the point charge is therefore

\[
E = \frac{\alpha q}{(4\pi \epsilon_0)^2 r^5} \left[ 2 \cos \theta \hat{r} + \sin \theta \hat{\theta} \right]
\]  

(5)

where \( \hat{r} \) points along the line from the atom to the point charge. The point charge is thus located at \( \theta = 0 \) so the force on the charge due to the dipole is

\[
F = \frac{2\alpha q^2}{(4\pi \epsilon_0)^2 r^5}
\]  

(6)