Suppose we have a coaxial cable with an inner conducting core of radius \( a \) and an outer conducting cylinder of radius \( c \). Part of the space between the conductors is filled with a linear dielectric with dielectric constant \( \varepsilon_r \) which extends from radius \( b \) to \( c \).

We can use an analysis similar to that which we used in working out the capacitance of a coaxial cable earlier. If we place a surface charge density of \( \sigma \) on the inner conductor, then using the analysis from before, the potential in the region \( a < r < b \) is

\[
V = \frac{a\sigma}{\varepsilon_0} \ln \frac{r}{a} \quad (1)
\]

For the region \( b < r < c \), the electric field is reduced by a factor of \( \varepsilon_r \) so in this region we have for the potential relative to radius \( b \):

\[
V = \frac{a\sigma}{\varepsilon_0 \varepsilon_r} \ln \frac{r}{b} \quad (2)
\]

The total potential difference between the two conductors is then

\[
V_{tot} = \frac{a\sigma}{\varepsilon_0} \ln \frac{b}{a} + \frac{a\sigma}{\varepsilon_0 \varepsilon_r} \ln \frac{c}{b} \quad (3)
\]

\[
= \frac{a\sigma}{\varepsilon_0} \left[ \ln \frac{b}{a} + \frac{1}{\varepsilon_r} \ln \frac{c}{b} \right] \quad (4)
\]

The capacitance per unit length of the cable is found from \( C = Q/V \). The charge per unit length is \( Q = 2\pi a\sigma \), so we get

\[
C = \frac{2\pi a\sigma}{\frac{a\sigma}{\varepsilon_0} \left[ \ln \frac{b}{a} + \frac{1}{\varepsilon_r} \ln \frac{c}{b} \right]} \quad (5)
\]

\[
= \frac{2\pi \varepsilon_0}{\ln \frac{b}{a} + \frac{1}{\varepsilon_r} \ln \frac{c}{b}} \quad (6)
\]