BIOT-SAVART LAW: A COUPLE MORE EXAMPLES

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Here are a couple more examples of calculating the magnetic field due to a steady current in a wire.

For the first configuration (bear with me as my picture drawing skills aren’t up to much so I’ll describe this in words), we have a loop defined as follows.

Starting at $x = a, y = 0$, draw a circular arc of radius $a$ through an angle of $\pi/2$ up to $x = 0, y = a$. Then draw a line out to $x = 0, y = b$, then another circular arc of radius $b$ back down to $x = b, y = 0$, then finally join this up with a line to $x = a, y = 0$ to complete the circuit. Find the field at the centre of the circles defining the arcs.

We apply the Biot-Savart law

$$
\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{I} \times (\mathbf{r} - \mathbf{r'})}{|\mathbf{r} - \mathbf{r}'|^3} \, dl' 
$$

(1)

where $\mathbf{r}'$ is a location on the wire and $\mathbf{r}$ is the point at which you want to determine the magnetic field. Along the two radial segments $\mathbf{I}$ is parallel to $\mathbf{r} - \mathbf{r}'$ so the cross product is zero, thus the only contributions come from the two arcs.

From the earlier post, we saw that the field due to a complete circular loop of radius $R$ is, where the current flows in a counter-clockwise direction as seen from above

$$
\mathbf{B} = \hat{z} \frac{I\mu_0}{2R} 
$$

(2)

Each arc is $\frac{1}{4}$ of a circle, and the current flows in the opposite direction in the outer arc, so the net field is

$$
\mathbf{B} = \hat{z} \frac{I\mu_0}{8a} - \hat{z} \frac{I\mu_0}{8b} = \hat{z} \frac{I\mu_0}{8} \left( \frac{1}{a} - \frac{1}{b} \right) 
$$

(3)

(4)
Now consider a semi-circular arc of radius $R$ extending from $\theta = +\frac{\pi}{2}$ to $\theta = -\frac{3\pi}{2}$. Join each end of this arc to a horizontal wire extending to infinity in the $x$ direction (so we have infinite wires along the lines $y = \pm R$, starting at the $y$ axis and extending out to positive infinity in the $x$ direction). The current flows in along the bottom wire and out along the top wire. We want the field at the centre of circle defining the semi-circular arc.

The field due to the semi-circle is (negative since the current is flowing clockwise)

$$B_s = -\hat{z} \frac{I \mu_0}{4R}$$  \hspace{1cm} (5)$$

For the two straight segments, we can use the same approach as in the earlier post. For the bottom wire, we get

$$B_b = -\hat{z} \frac{I \mu_0}{4\pi} \int_0^\infty \frac{R}{(R^2 + x^2)^{3/2}} dx$$  \hspace{1cm} (6)$$

$$= -\hat{z} \frac{I \mu_0}{4\pi R}$$  \hspace{1cm} (7)$$

By symmetry, the top wire contributes the same amount, so the total field is

$$B = B_s + 2B_b$$  \hspace{1cm} (8)$$

$$= -\hat{z} \frac{I \mu_0}{4R} \left( 1 + \frac{2}{\pi} \right)$$  \hspace{1cm} (9)$$