MAGNETIC VECTOR POTENTIAL: SHEET OF CURRENT

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Another example of calculating the magnetic vector potential in a case where the current extends to infinity. We consider a uniform sheet of current in the $xy$ plane, carrying surface current density $K \hat{x}$. Using the same argument as in the case of a slab of current, the magnetic field due to this current is

$$B = \begin{cases} -\frac{\mu_0}{2} K \hat{y} & z > 0 \\ \frac{\mu_0}{2} K \hat{y} & z < 0 \end{cases} \quad (1)$$

Note that the field is independent of the distance from the sheet of current.

We can work out the potential by applying Stokes’s theorem. From the definition of $A$, we know that $\nabla \times A = B$, so if we define a closed loop then we have

$$\oint A \cdot dl = \int_B \cdot da \quad (2)$$

where the integral on the LHS is a line integral around the loop and the integral on the right is over the area $A$ enclosed by the loop. For a flat loop, the direction of integration on the LHS is such that the right hand rule gives a vector pointing in the same direction as $da$ on the RHS.

We need to be careful in defining directions for the area and line integrals to get the signs right. We’ll take as our area $A$ a rectangle above the $xy$ plane and parallel to the $xz$ plane. The sides of the rectangle parallel to the $xy$ plane are of length $1$, with the lower side at a distance $a$ and the upper side at a distance $b$ from this plane. The normal to the area points in the $\hat{y}$ direction. Since $B$ is uniform everywhere above the plane, we get

$$\int_A B \cdot da = -\frac{\mu_0}{2} K (b - a) \quad (3)$$

For the line integral, the path around the rectangle is clockwise when looking in the $+y$ direction, and by symmetry, the integral of $A \cdot dl$ along the vertical sides cancels out, so
\[ \oint A \cdot dl = A_x(b) - A_x(a) \]  

(4)

Comparing these two, a reasonable candidate is

\[ A = -\frac{\mu_0}{2} Kz \hat{x} \]  

(5)

We can check this by finding the div and curl, as usual:

\[ \nabla \cdot A = 0 \]  

(6)

\[ \nabla \times A = -\frac{\mu_0}{2} K \hat{y} \]  

(7)

Below the sheet of current, the sign is reversed so we have

\[ A = \frac{\mu_0}{2} Kz \hat{x} \]  

(8)