HELMHOLTZ COIL

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The magnetic field due to a circular current loop of radius $R$ and current $I$ a distance $z$ above the centre of the loop is (Griffiths example 5.6):

$$ B = \frac{\mu_0 I R^2}{2} \frac{1}{[R^2 + z^2]^{3/2}} \quad (1) $$

If we take two such current loops a distance $d$ apart and set $z = 0$ to be halfway between them, then the net field on the axis is, by the principle of superposition:

$$ B = \frac{\mu_0 I R^2}{2} \left[ \frac{1}{[R^2 + (z - \frac{d}{2})^2]^{3/2}} + \frac{1}{[R^2 + (z + \frac{d}{2})^2]^{3/2}} \right] \quad (2) $$

The derivative of $B$ is

$$ \frac{\partial B}{\partial z} = \frac{\mu_0 I R^2}{2} \left( -\frac{3}{2} \frac{2 z - d}{\left( R^2 + (z - \frac{d}{2})^2 \right)^{5/2}} - \frac{3}{2} \frac{2 z + d}{\left( R^2 + (z + \frac{d}{2})^2 \right)^{5/2}} \right) \quad (3) $$

The derivative is always zero at $z = 0$.
Now the second derivative is

$$ \frac{2}{\mu_0 I R^2} \frac{\partial^2 B}{\partial z^2} = \frac{15}{4} \frac{(2 z - d)^2}{\left( R^2 + (z - \frac{1}{2} d)^2 \right)^{7/2}} - \frac{3}{2} \frac{R^2 + (z - \frac{1}{2} d)^2}{\left( R^2 + (z - \frac{1}{2} d)^2 \right)^{5/2}} \quad (4) $$

$$ + \frac{15}{4} \frac{(2 z + d)^2}{\left( R^2 + (z + \frac{1}{2} d)^2 \right)^{7/2}} - \frac{3}{2} \frac{R^2 + (z + \frac{1}{2} d)^2}{\left( R^2 + (z + \frac{1}{2} d)^2 \right)^{5/2}} $$

At $z = 0$ this becomes
\[
\frac{\partial^2 B}{\partial z^2} = \frac{\mu_0 IR^2}{2} \left( \frac{15}{2} \left( \frac{d^2}{R^2 + \frac{1}{4} d^2} \right)^{7/2} - \frac{6}{(R^2 + \frac{1}{4} d^2)^{5/2}} \right) 
\] (5)

We can make this derivative zero as well if we choose

\[ d = R \] (6)

Under this condition, the field at the centre is

\[ B(0) = \frac{8I\mu_0}{5\sqrt{5}R} \] (7)

Such a pair of circular loops is known as a *Helmholtz coil* and is used to produce a localized region of a relatively constant magnetic field. In fact, the third derivative also turns out to be zero at \( z = 0 \), so the first non-zero derivative is the fourth derivative which (if you’re interested) comes out to

\[ \frac{\partial^4 B}{\partial z^4} (z = 0) = -\frac{27648\sqrt{5}}{3125} \frac{\mu_0 I}{R^5} \] (8)