CHARGING AND DISCHARGING A CAPACITOR: GENERAL CASE

Link to: physicspages home page.
To leave a comment or report an error, please use the auxiliary blog.
Post date: 18 Sep 2013.

The problem of charging and discharging a capacitor can be generalized to two chunks of conducting material of arbitrary shape embedded within a medium with conductivity $\sigma$. First, we can work out the resistance between the two conductors. Suppose at some instant in time the potential difference between the two conductors is $V$, and the current flowing between them is $I$. This current is the surface integral of the current density over some area that encloses one of the conductors; let’s make it the conductor with a net positive charge $Q$ on its surface. That is

$$I = \int \mathbf{J} \cdot d\mathbf{a}$$

(1)

By our definition of conductivity, we’re taking $\mathbf{J} = \sigma \mathbf{E}$ so

$$I = \sigma \int \mathbf{E} \cdot d\mathbf{a} = \frac{\sigma Q}{\epsilon_0}$$

(2)

using Gauss’s law. The capacitance of the system is given by $C = Q/V$ and Ohm’s law says that $V = IR$ so

$$I = \frac{V}{R} = \frac{Q}{CR} = \frac{\sigma Q}{\epsilon_0}$$

(3)

$$R = \frac{\epsilon_0}{\sigma C}$$

(4)

Given the resistance between the conductors, the problem of finding the charge, current and potential as functions of time reduces to that in the previous post. So we have
\[ Q(t) = CV_0 e^{-t/RC} \]  
\[ I(t) = \dot{Q}(t) = -\frac{CV_0 \sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0} \]  
\[ V(t) = -I(t) R = \frac{RCV_0 \sigma}{\epsilon_0} e^{-\sigma t/\epsilon_0} = V_0 e^{-\sigma t/\epsilon_0} \]

Charging the capacitor with a battery of fixed potential \( V_0 \) gives the same results as in the previous post with \( RC = \epsilon_0/\sigma \):

\[ Q(t) = CV_0 \left( 1 - e^{-\sigma t/\epsilon_0} \right) \]  
\[ I(t) = \dot{Q}(t) = \frac{V_0}{R} e^{-\sigma t/\epsilon_0} = \frac{\sigma CV_0}{\epsilon_0} e^{-\sigma t/\epsilon_0} \]  
\[ V(t) = \frac{Q(t)}{C} = V_0 \left( 1 - e^{-\sigma t/\epsilon_0} \right) \]

The voltage across the resistor is \( V_R(t) = V_0 - V(t) = V_0 e^{-\sigma t/\epsilon_0} = I(t) R. \)

PINGBACKS

Pingback: Momentum in a capacitor
Pingback: Angular momentum in electromagnetic fields