Here’s an example of using Ohm’s law to calculate the resistance of a system with variable conductivity. We have a coaxial cylinder of length \( L \) with an inner cylinder of radius \( a \) and an outer cylinder of radius \( b \). The conductivity is \( \sigma(r) = k/r \) where \( k \) is a constant. If the two cylinders are held at a constant potential difference, then the current passing between them is also constant \( I \). We can get \( I \) by integrating over a cylindrical surface between the two cylinders:

\[
I = \sigma(r) \oint E \cdot da
\]  

(1)

By symmetry (ignoring end effects, or else considering a length \( L \) within a much longer cylinder), \( E \) is radial and can depend at most on \( r \), so if we choose an integration cylinder of a fixed radius \( r \) then

\[
I = \frac{k}{r} 2\pi r EL = 2\pi kL E
\]  

(2)

\[
E = \frac{I}{2\pi kL}
\]  

(3)

The field is therefore constant between the cylinders, which means the potential difference between the cylinders is just

\[
V = E(b-a) = \frac{I}{2\pi kL}(b-a)
\]  

(4)

giving a resistance of (using Ohm’s law \( V = IR \)):

\[
R = \frac{b-a}{2\pi kL}
\]  

(5)