ENERGY IN A TOROIDAL SOLENOID

Here’s another simple example of calculating the energy in the magnetic field. This time we’ll look at a toroidal solenoid with $N$ turns carrying a current $I$. The field is given in Griffiths’s example 5.10:

$$\mathbf{B} = \frac{\mu_0 N I \hat{\phi}}{2\pi r}$$ (1)

This formula applies to a torus with an arbitrary cross section, but we’ll look at a rectangular cross section here. The inner radius is $a$, outer radius $b$ and height $h$. The energy is

$$W_B = \frac{1}{2\mu_0} \int B^2 d^3 r$$ (2)

$$= \frac{1}{2\mu_0} \left( \frac{\mu_0 NI}{2\pi} \right)^2 \int_a^b \frac{2\pi rh}{r^2} dr$$ (3)

$$= \frac{\mu_0 N^2 I^2 h}{4\pi} \ln \frac{b}{a}$$ (4)

Since $W_B = \frac{1}{2} LI^2$ we can use this formula to find the inductance of the torus.

$$L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$$ (5)

which agrees with Griffiths’s equation 7.27 in example 7.11.