MAGNETIC MONOPOLE FORCE LAW

If magnetic monopoles did exist, we could consider the behaviour of magnetic charges \( q_m \). We could postulate a Coulomb’s law for them:

\[
\mathbf{F} = \frac{\mu_0 q_m q_m' (\mathbf{r}_2 - \mathbf{r}_1)}{4\pi |\mathbf{r}_2 - \mathbf{r}_1|^3} \tag{1}
\]

Griffiths sets the problem of ‘working out’ the force law for a magnetic charge passing through electric and magnetic fields. This strikes me as rather difficult, since the Lorentz force law for electric charges is essentially a postulate of the theory (confirmed by experiment); it was never derived. The simplest equivalent magnetic force law would seem to be something like

\[
\mathbf{F} = q_m \mathbf{B} + q_m \mathbf{v} \times \mathbf{E} \tag{2}
\]

We do need to check the units, however. First, what are the units of a magnetic charge \( q_m \)? The units of \( \mu_0 \) are m kg s\(^{-2}\) A\(^{-2}\) so the Coulomb force law gives us

\[
\text{kg m s}^{-2} = \text{m kg s}^{-2} \text{A}^{-2} [q_m]^2 \text{m}^{-2} \tag{3}
\]

so the units of \( q_m \) must be A m = Coulomb m s\(^{-1}\), that is, charge times velocity. How does this fit with our proposed Lorentz-like force law? The units of \( \mathbf{B} \) are kg m s\(^{-2}\) A\(^{-1}\) m\(^{-1}\), so \( q_m \mathbf{B} \) has the units of force, so that term seems OK.

For the second term, we know that electric charge times electric field give force, so that means that \( q_m \mathbf{v} \times \mathbf{E} \) has the units of force times velocity squared, so we need to divide by some constant velocity squared. Since we know that \( \mu_0 \varepsilon_0 = 1/c^2 \) this seems a logical choice, so we can propose

\[
\mathbf{F} = q_m \mathbf{B} + \mu_0 \varepsilon_0 q_m \mathbf{v} \times \mathbf{E} = q_m \mathbf{B} + \frac{q_m}{c^2} \mathbf{v} \times \mathbf{E} \tag{4}
\]

In fact, the commonly accepted form is
\[ \mathbf{F} = q_m \mathbf{B} - \frac{q_m}{c^2} \mathbf{v} \times \mathbf{E} \] (5)

I could not find a 'derivation' of this result; rather it seems that this form is adopted to make this force law invariant under a [duality transformation](#).