CHANGING A PARTICLE’S SPEED IN A CYCLOTRON

Link to: [physicspages home page](http://www.physicspages.com/).
To leave a comment or report an error, please use the [auxiliary blog](http://www.physicspages.com/auxiliaryblog).
Post date: 11 Nov 2013.

We can now revisit the problem of a charged particle in a cyclotron field. Suppose we start with a charged particle of mass \( m \) and charge \( q \) at rest a distance \( R \) from the axis of the cyclotron, and we want to increase the speed of the particle in its circular orbit while keeping it at the same radius (such a device is called a betatron). We can do this by varying the magnetic field \( B(t) \) such that the cyclotron relation is always satisfied:

\[
qvB = \frac{mv^2}{R} \quad (1)
\]

\[
B = \frac{mv}{qR} \quad (2)
\]

Taking the time derivative, we get

\[
\dot{B} = \frac{m\dot{v}}{qR} \quad (3)
\]

The changing magnetic field induces a circumferential electric field \( E \), and since this field is parallel to the particle’s direction of motion it will act as the force that accelerates the particle. From Newton’s law, \( F = m\dot{v} = qE \), so

\[
\dot{B} = \frac{E}{R} \quad (4)
\]

If we assume the cyclotron has cylindrical symmetry, then we can integrate this equation along the particle’s orbit, along which both fields are constant in magnitude. That is

\[
\int B_c d\ell = \int \frac{E}{R} d\ell \quad (5)
\]

\[
\int E d\ell = R \int \dot{B}_c d\ell = 2\pi R^2 \dot{B}_c \quad (7)
\]
This equation applies on the circumference of the orbit only (not in the interior of the orbit), so we’ve added a suffix $c$ to $B_c$ to emphasize this point.

From Faraday’s law in integral form, we have also that the integral of the electric field is given by the change in flux:

$$\oint E \, dl = -\frac{d\Phi}{dt}$$  \hspace{1cm} (8)  

$$= - \int B \cdot da$$  \hspace{1cm} (9)  

where now we are integrating over all points within the orbit.

If we start with the particle at rest in zero field, then we have

$$2\pi R^2 B_c = - \int B \cdot da = \pi R^2 \dot{B}$$  \hspace{1cm} (10)  

where $\dot{B}$ is the average field across the orbit.

A word about the signs here. Suppose the magnetic field points in the $-z$ direction (as shown in Fig. 7.52 in Griffiths), and we take the area vector to point in the $+z$ direction. Further, if the magnetic field is increasing in magnitude, then $\dot{B}$ points towards $-z$ as well. Thus $\dot{B} \cdot da < 0$, giving the sign shown.

We can integrate both sides to some time $t$ to get

$$\dot{B}_c(t) = \frac{1}{2} \dot{B}(t)$$  \hspace{1cm} (11)  

Thus we can speed up (or slow down, by decreasing the field) the particle by keeping the average field equal to twice the field at the radius of the orbit.