For the sinusoidal wave on a string, let’s assume that the wave travels in the +z direction and then define x and y axes in the usual way. We can produce the wave by shaking the string in the xz plane, in which case there is no component of the wave in the y direction, or by shaking the string in the yz plane, in which case there is no motion in the x direction. Or we could shake the string in some other plane intermediate between xz and xy, as long as that plane contains the z axis.

If the x axis points upwards and the y axis points horizontally, then a wave moving in the xz plane can be said to be vertically polarized, while a wave moving in the yz plane is horizontally polarized. (These terms aren’t really technical terms, but you get the idea.) Any transverse wave whose motion is restricted to a plane is linearly polarized. We can represent this in vector notation by writing

\[ f(z,t) = A e^{i(kz - \omega t)} \hat{n} \]

where \( \hat{n} \) is a unit vector perpendicular to the z axis and parallel to the plane of polarization. Thus for a wave moving in the xz plane, \( \hat{n} = \hat{x} \) and so on.

If \( \theta \) is the angle between \( \hat{n} \) and \( \hat{x} \), then in general

\[ \hat{n} = \cos \theta \hat{x} + \sin \theta \hat{y} \]

and a wave linearly polarized in the \( \hat{n} \) direction is given by

\[ f(z,t) = A e^{i(kz - \omega t)} (\cos \theta \hat{x} + \sin \theta \hat{y}) \]

More generally, we can have a wave that is the vector sum of a vertically and horizontally polarized wave:

\[ f(z,t) = A_x e^{i(kz - \omega t)} \cos \theta \hat{x} + A_y e^{i(kz - \omega t)} \sin \theta \hat{y} \]

where the complex amplitudes are given by
with $i = x, y$. The real amplitudes $a_x$ and $a_y$ must be the same for both components in order for the polarization to be in the $\hat{n}$ plane.

For a wave to be linearly polarized, the phases of the two component waves must also be equal, so that $\delta_x = \delta_y = \delta$. In that case, the components of the wave at a fixed location $z = z_0$ vary with time according to

$$f_x(z_0, t) = ae^{i(kz_0 - \omega t + \delta)} \cos \theta \hat{x}$$

$$f_y(z_0, t) = ae^{i(kz_0 - \omega t + \delta)} \sin \theta \hat{y}$$

Since it is only the real parts of these equations that are physically meaningful, we see that the time dependence is the same in both cases and is $\cos (kz_0 - \omega t + \delta)$. Thus the horizontal and vertical components oscillate in phase, so that the wave remains in the $\hat{n}$ plane.

Now suppose that $\delta_y = \pi/2$ and $\delta_x = 0$ so that the two components are out of phase. Then the real components of the wave are

$$x = \Re(f_x) = a \cos (kz_0 - \omega t) \cos \theta$$

$$y = \Re(f_y) = a \cos \left( kz_0 - \omega t + \frac{\pi}{2} \right) \sin \theta$$

$$= -a \sin (kz_0 - \omega t) \sin \theta$$

In this case, the point on the string at position $z = z_0$ follows a curve with equation

$$\frac{x^2}{(a \cos \theta)^2} + \frac{y^2}{(a \sin \theta)^2} = 1$$

which is the equation of an ellipse. In this case, the wave is elliptically polarized. If $\theta = \frac{\pi}{4}$ then we get a circle and the wave is circularly polarized.

To see which direction the string moves, try a couple of values of $t$ at $z_0 = 0$. For $t = 0$, $x(0,0) = a \cos \theta$, $y(0,0) = 0$. Then for $t = \pi/2\omega$ we have $x(0, \pi/2\omega) = 0$, $y(0, \pi/2\omega) = a \sin \theta$. Thus if $\theta$ increases counterclockwise as we look down the $z$ axis towards the origin, the point is rotating counterclockwise. The shape of the string at any given instant of time is a helix (either elliptical or circular, depending on $\theta$). We can generate this by shaking the end of the string in an ellipse or circle.

If we set $\delta_y = -\pi/2$ then
and this time, a point on the string moves clockwise around the $z$ axis.