FIELDS OF A POINT CHARGE MOVING IN ONE DIMENSION

Here’s a simple example of calculating the fields due to a moving point charge. Suppose we have a charge constrained to move on the \(x\) axis in the \(+x\) direction, so that \(v = v \hat{x}\).

The fields are

\[
E(r, t) = \frac{q \tau}{4\pi\varepsilon_0 (r \cdot u)^3} \left[ (c^2 - v^2) u + r \times (u \times a) \right] \tag{1}
\]

\[
B(r, t) = \frac{1}{c} \hat{r} \times E(r, t) \tag{2}
\]

where

\[
r \equiv r - w(t_r) \tag{3}
\]

\[
u \equiv c \hat{r} - v \tag{4}
\]

and \(w(t_r)\) is the particle’s position at the retarded time. If the observer is to the right of the particle then

\[
r = +r \hat{x} \tag{5}
\]

\[
u = (c - v) \hat{x} \tag{6}
\]

Since the motion is constrained to the \(x\) axis, any velocity and acceleration must be parallel, so \(u \times a = 0\) and we have

\[
E = \frac{q \tau}{4\pi\varepsilon_0 (r \cdot u)^3} (c^2 - v^2) u \tag{7}
\]

\[
= \frac{q (c^2 - v^2)}{4\pi\varepsilon_0 (c - v)^3} c \hat{x} \tag{8}
\]

\[
= \frac{q (c + v)}{4\pi\varepsilon_0 (c - v) c^2} \hat{x} \tag{9}
\]
Because \( \hat{\mathbf{r}} \) is parallel to \( \mathbf{E} \), \( \mathbf{B} = 0 \) from [2].

If the observer is to the left of the charge, then

\[
\begin{align*}
\mathbf{r} &= -r \hat{x} \\
\mathbf{u} &= -(c + v) \hat{x} \\
\mathbf{E} &= \frac{q r}{4 \pi \varepsilon_0 (\mathbf{r} \cdot \mathbf{u})^3} (c^2 - v^2) \mathbf{u} \\
&= -\frac{q (c^2 - v^2)}{4 \pi \varepsilon_0 (c + v)^3} \frac{c + v}{v^2} \hat{x} \\
&= -\frac{q (c - v)}{4 \pi \varepsilon_0 (c + v) r^2} \hat{x} \\
\mathbf{B} &= 0
\end{align*}
\]