RELATIONS AMONG CHARGE, CURRENT, POTENTIALS AND FIELDS

Link to: physicspages home page.
To leave a comment or report an error, please use the auxiliary blog.
Post date: 30 Nov 2014.

With a complete theory of electrodynamics, it’s useful to summarize the relations between the sources of the fields (charge density $\rho$ and current density $J$), the potentials $V$ and $A$ and the fields $E$ and $B$.

Starting with $\rho$ and $J$, we can calculate the retarded potentials

$$V(r,t) = \frac{1}{4\pi \epsilon_0} \int \frac{\rho(r',t_r)}{r} d^3r'$$

$$A(r,t) = \frac{\mu_0}{4\pi} \int \frac{J(r',t_r)}{r} d^3r'$$

Or we can get the fields from Jefimenko’s equations for the electric and magnetic fields.

$$E(r,t) = \frac{1}{4\pi\epsilon_0} \int \left( \frac{\rho(r',t_r)}{r^2} \hat{r} + \frac{\dot{\rho}(r',t_r)}{cr} \hat{r} - \frac{\hat{J}(r',t_r)}{c^2r} \right) d^3r'$$

$$B(r,t) = \frac{\mu_0}{4\pi} \int \left[ \frac{\hat{J}(r',t_r) \times \hat{r}}{cr} + J(r',t_r) \times \frac{\hat{r}}{r^2} \right] d^3r'$$

Inverting the procedure, we can get the sources from the potentials, although we need to know the gauge we’re using. In the Lorentz gauge we have

$$\square^2 V = -\frac{\rho}{\epsilon_0}$$

$$\square^2 A = -\mu_0 J$$

From the fields, we can use Maxwell’s equations
\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \quad (7) \]
\[ \nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J} \quad (8) \]

Finally, from the potentials we can get the fields

\[ \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \quad (9) \]
\[ \mathbf{B} = \nabla \times \mathbf{A} \quad (10) \]