LORENTZ TRANSFORMATIONS AND SIMULTANEITY

Here's a simple example of using the Lorentz transformations. Suppose that two events occur simultaneously in the Earth frame, a distance 500 km apart. An observer in a (very fast) plane travelling at $\frac{12}{13} c$ along the line joining the events passes the first event $A$ such that their respective coordinate origins coincide at that event. At what time does the plane observer think event $B$ occurs?

The Lorentz transformations are

\begin{align*}
\bar{x} &= \gamma(x - vt) \quad (1) \\
\bar{t} &= \gamma \left( t - \frac{xv}{c^2} \right) \quad (2)
\end{align*}

Event $A$ occurs at the same time $t = \bar{t} = 0$ in both systems. For event $B$, $x = 500$ km and $t = 0$, so

\begin{align*}
\bar{t} &= \gamma \left( -\frac{500v}{c^2} \right) \\
\gamma &= \frac{13}{5} \quad (4)
\end{align*}

\begin{align*}
\bar{t} &= -\frac{13}{5} \left( \frac{12c \times 500}{c^2} \right) \\
&= -\frac{1200}{c} \\
&= -\frac{1200 \text{ km}}{3 \times 10^5 \text{ km s}^{-1}} \quad (6)
\end{align*}

\begin{align*}
&= -4 \times 10^{-3} \text{ s} \quad (7)
\end{align*}

Thus the plane observer thinks that $B$ occurs 4 milliseconds before $A$. 

\[1\]