FORCE IN RELATIVITY

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Force can be defined in relativity as the derivative of the spatial components of four-momentum with respect to ordinary (non-proper) time:

\[ F = \frac{dp}{dt} \]  

(1)

Superficially, this looks the same as Newton’s second law, but in fact the formula for force is a bit more complex when written out in full. Using the definition of momentum, we have

\[ F = \frac{d}{dt} [\gamma mu] \]  

(2)

where

\[ \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \]  

(3)

We have

\[ \frac{d \gamma}{dt} = \frac{1}{2c^2 (1 - u^2/c^2)^{3/2}} \frac{d (u \cdot u)}{dt} \]  

(4)

\[ = \frac{\gamma^3}{2c^2} (2u \cdot a) \]  

(5)

\[ = \frac{u \cdot a}{c^2 (1 - u^2/c^2)^{3/2}} \]  

(6)

where \( a \equiv \ddot{u} \) is the acceleration.

Returning to (2) we have

1
\[ F = m \mathbf{u} \frac{d\gamma}{dt} + \gamma m \mathbf{\dot{u}} \quad (7) \]

\[ = \frac{m (\mathbf{u} \cdot \mathbf{a}) \mathbf{u}}{c^2 (1 - u^2/c^2)^{3/2}} + \frac{m \mathbf{a}}{\sqrt{1 - u^2/c^2}} \quad (8) \]

\[ = \frac{m}{\sqrt{1 - u^2/c^2}} \left[ \mathbf{a} + \frac{(\mathbf{u} \cdot \mathbf{a}) \mathbf{u}}{c^2 - u^2} \right] \quad (9) \]

This formula reduces to the familiar \( F = ma \) in the limit of small \( \mathbf{u} \). However, if we wish to retain a fixed acceleration as \( u \to c \), the required force becomes infinite. Or looked at another way, if we want the force to remain finite as \( u \to c \), the acceleration must drop to zero. In other words, it’s impossible to accelerate an object with a non-zero rest mass to the speed of light.

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