GAMMA MATRICES IN DIRAC-PAULI REPRESENTATION

The gamma matrices that appear in the Dirac equation can, for the most part, be used without writing them down as explicit $4 \times 4$ matrices. To examine explicit solutions of the Dirac equation, however, it’s useful to have a particular form for the matrices. One such form is the Dirac-Pauli representation, in which

$$\gamma^0 = \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix}$$

$$\gamma^i = \begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix}$$

where the $\sigma^i$ are the Pauli matrices

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}; \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Each of the entries in [1] and [2] is a $2 \times 2$ matrix, while the entries in the Pauli matrices are ordinary numbers. We can verify by direct calculation that this representation of the $\gamma^\mu$'s obey the required anticommutation relations and other relations, which we reproduce here for convenience:
\( \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \)  
(4)

\( \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \)  
(5)

\( (\gamma^0)^2 = 1 \)  
(6)

\( (\gamma^i)^2 = -1 \)  
(7)

\( \text{Tr} \gamma^\mu = 0 \)  
(8)

\( \text{Tr} \gamma^5 = 0 \)  
(9)

\( \{\gamma^\mu, \gamma^5\} = 0 \)  
(10)

\( (\gamma^\mu)^\dagger = \gamma^0\gamma^\mu\gamma^0 \)  
(11)

We can work out \( \gamma^5 \) in this representation by direct multiplication.

\[
\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3
\]  
(12)

\[
= i \begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} 0 & \sigma^1 \\ -\sigma^1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix}
\]  
(13)

\[
= i \begin{bmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix}
\]  
(14)

\[
= i \begin{bmatrix} -\sigma^1\sigma^2 & 0 \\ 0 & \sigma^1\sigma^2 \end{bmatrix} \begin{bmatrix} 0 & \sigma^3 \\ -\sigma^3 & 0 \end{bmatrix}
\]  
(15)

\[
= i \begin{bmatrix} 0 & \sigma^1\sigma^2\sigma^3 \\ -\sigma^1\sigma^2\sigma^3 & 0 \end{bmatrix}
\]  
(16)

By multiplying out \( 3 \) we find

\[
\sigma^1\sigma^2\sigma^3 = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = iI
\]  
(17)

so we have

\[
\gamma^5 = i \begin{bmatrix} 0 & -iI \\ -iI & 0 \end{bmatrix} = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}
\]  
(18)

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