WICK’S THEOREM - NO EQUAL TIME CONTRACTIONS

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Wick’s theorem states that we can express a time-ordered product of operators as a series of terms involving normal ordered products and contractions, with the result

\[
\mathcal{T} [ABC \ldots XYZ] = \mathcal{N} ABC \ldots XYZ + \mathcal{N} ABC \ldots XYZ + \mathcal{N} ABC \ldots XYZ + \mathcal{N} ABCD \ldots XYZ + \mathcal{N} ABCD \ldots XYZ + \mathcal{N} ABC \ldots WXYZ + \ldots
\]

all higher order contractions

This theorem is to be applied to the calculation of the S-matrix, which is defined as the limit at infinite time of the evolution operator

\[
S \equiv \lim_{t \to \infty} U(t) = 1 + \sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \int d^4 x_1 \int d^4 x_2 \ldots \int d^4 x_n
\]

where we’ve written the integrals as being done over the hamiltonian densities, so all integrals are over all four spacetime coordinates.

In practice, each of the interaction hamiltonian terms \( \mathcal{H}_I(x) \) in the time-ordered product will usually be a product of several field operators such as \( \psi \) and \( \overline{\psi} \) for fermions and \( \phi \) for scalars. Since each \( \mathcal{H}_I \) is evaluated at a specific time, the time arguments of all its constituent fields will be the same. As a result, if for example

\[
\mathcal{H}_I(x) = A(x) B(x) C(x)
\]
we’re faced with a time-ordered product of the form

\[ \mathcal{T} [H_i(x_1) H_i(x_2) \ldots H_i(x_n)] = \mathcal{T} [:ABC(x_1): :ABC(x_2) : : :ABC(x_n):] \]

(12)

As all field operators within each normal-ordered product are at the same spacetime point, their commutators or anticommutators can be infinite, which we need to avoid. To avoid this, an infinitesimal time \( \epsilon_r \) is added to each creation operator and subtracted from each annihilation operator within the fields in each normal-ordered product. This has the effect of placing all the creation operators to the left of all annihilation operators when time ordering is performed. That ordering is automatically normal-ordered as well, so we don’t need to apply Wick’s theorem to the fields within each normal-ordered product, as they are already time-ordered. The net result is that, whenever we are faced with a time-ordered product containing fields that are at equal times, we don’t do any contractions over pairs of such fields. As stated by L&P, this is given by

\[ \mathcal{T} [::ABC(x_1)::ABC(x_2):: : : :ABC(x_n)::] = \mathcal{T} [(ABC(x_1))(ABC(x_2)) \ldots (ABC(x_n))] \]

(13)

where “no e.t.c.” means “no equal-time contractions”.

This again seems like a bit of a kludge (along the lines of normal ordering, which seemed quite kludgy in the first place) but I suppose it is justified by the fact that the results do seem to agree with experiments.

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