FERMION SCATTERING - FOURTH-ORDER FEYNMAN DIAGRAMS

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In their section 6.8, L&P describe a technique for evaluating some of the internal parts of certain Feynman diagrams. As an example, they look at some of the fourth-order diagrams for electron-electron scattering. The diagrams they consider are these:

We won’t go through the calculation of the S-matrix elements from these diagrams again here. Rather, we’re interested in what other fourth-order diagrams there are.

The fourth order terms in electron-electron scattering using the Yukawa hamiltonian are all derived from this term:

\[ \langle \psi \psi \phi \rangle_{x_1} \langle \bar{\psi} \psi \phi \rangle_{x_2} \langle \bar{\psi} \psi \phi \rangle_{x_3} \langle \bar{\psi} \psi \phi \rangle_{x_4} \]  \(1\)

Since the scattering process we’re studying has two incoming fermions and two outgoing fermions, there must be two \( \bar{\psi} \) fields and two \( \psi \) fields.

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Figure 2. Fermion loop connected with scalar contractions.

present in any term that gives a non-zero matrix element. All other fields
must be contracted, giving propagators. Therefore, we must have two \( \phi \phi \)
contractions and two \( \bar{\psi} \psi \) contractions. The diagrams shown in [1] are ob-
tained by contracting the fermions over 3 coordinates, leaving the fourth
coordinate to represent an incoming fermion going straight to an outgoing
fermion. For example, the top-left diagram is obtained from

\[
(\bar{\psi} \psi \phi)_{x_1} (\bar{\psi} \psi \phi)_{x_2} (\bar{\psi} \psi \phi)_{x_3} (\bar{\psi} \psi \phi)_{x_4};
\]  

(2)

The other three diagrams are permutations of this.

The other possible diagrams can be obtained by considering how to group
the contractions. They are as follows.

First, we can contract the \( \bar{\psi} \) and \( \psi \) fields from two of the coordinates with
each other, leaving the other two coordinates uncontracted. This creates a
fermion loop. The \( \phi \) fields can then be contracted in two ways. First, we can
connect the fermion loop to the incoming and outgoing fermions by using
the two scalar contractions as in Fig. [2].

Second, we can create an isolated fermion loop, (Fig. [3]).

As L&P discuss with their Figure 6.3, the loop on the left is a vacuum-to-
vacuum diagram which contributes nothing to the scattering process. The
remaining part of the diagram on the right is equivalent to the second-order
term examined by L&P in their section 6.4. Both of these diagrams have
permutations which I won’t include as it gets tedious to draw the diagrams.
Finally, we can involve all four coordinates in the fermion contractions, giving Fig. 4.

In these diagrams, the two boson propagators carry momentum back and forth between the two fermions. Note that momentum is conserved at each vertex. For example, in the top left diagram, the vertex at the top right of the square results in an exiting fermion with momentum

\[ p_1 - k + p_2 + k - p_2' \]  

However, we know from momentum conservation that \( p_1 + p_2 = p_1' + p_2' \) so \( p_2 - p_2' = p_1' - p_1 \), so we have

\[ p_2 - p_2' = p_1' - p_1 \] (3)
\[ p_1 - k + p_2 + k - p'_2 = p'_1 \]  \hspace{1cm} (4)

which is the correct momentum for the fermion leaving the scattering at the top right of the diagram.