The complex scalar field considered by L&B in their Chapter 12 is invariant under the internal transformation
\[ \psi \rightarrow \psi' = e^{i\alpha} \psi \] (1)

They point out that this transformation can also be written using the unitary operator
\[ U(\alpha) = e^{iQ_{Nc}\alpha} \] (2)

where \( Q_{Nc} \) is the conserved number charge operator, which is defined in terms of the Noether charge \( Q_N \) by normal ordering:
\[ Q_{Nc} = -:\!Q_N\!:\ ] (3)

To verify that this unitary operator works correctly, we consider \( U^\dagger \psi U \) for an infinitesimal \( \alpha \). We have, keeping only terms up to first order in \( \alpha \)
\[ U^\dagger(\alpha) \psi U(\alpha) = (1 - iQ_{Nc}\alpha) \psi (1 + iQ_{Nc}\alpha) \] (4)
\[ = \psi - i\alpha [Q_{Nc}, \psi] \] (5)

We can now use the formula
\[ [Q_N, \psi] = -iD\psi \] (6)

where
\[ D\psi = \left. \frac{\partial \psi'}{\partial \alpha} \right|_{\alpha=0} = i\psi \] (7)

From (3) we have
\[ [Q_{Nc}, \psi] = -[Q_N, \psi] = -\psi \] (8)

so we have
\[ U^\dagger(\alpha) \psi U(\alpha) = \psi (1 + i\alpha) \] (9)
Taking $\alpha$ to be finite, we can apply this formula $n$ times and let $n \to \infty$ to get

$$U^\dagger (\alpha) \psi U (\alpha) = \lim_{n \to \infty} \psi \left( 1 + i \frac{\alpha}{n} \right)^n \quad (10)$$

$$= e^{i\alpha} \psi \quad (11)$$

Thus the unitary transformation is equivalent to the transformation $1$.

In case you’re worried that the normal ordering done to get from $Q_N$ to $Q_{Nc}$ messes up the relation $8$, we can see that this isn’t a problem by considering $Q_{Nc}$ and $\psi$ in their mode expansions. From L&B’s equations 12.5 and 12.15, we have

$$\psi (x) = \int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2E_p}} \left( a_p e^{-ip \cdot x} + b_p^\dagger e^{ip \cdot x} \right) \quad (12)$$

$$Q_{Nc} = \int d^3 q \left( n^{(a)}_{\mathbf{q}} - n^{(b)}_{\mathbf{q}} \right) \quad (13)$$

$$= \int d^3 q \left( a_{\mathbf{q}}^\dagger a_{\mathbf{q}} - b_{\mathbf{q}}^\dagger b_{\mathbf{q}} \right) \quad (14)$$

Taking the commutator explicitly and using

$$[a_p, a_{\mathbf{q}}^\dagger] = [b_p^\dagger, b_{\mathbf{q}}] = \delta (\mathbf{p} - \mathbf{q}) \quad (15)$$

with all other commutators being zero, we have

$$[Q_{Nc}, \psi] = \int \frac{d^3 p \, d^3 q}{(2\pi)^{3/2} \sqrt{2E_p}} \left\{ \left[ a_{\mathbf{q}}^\dagger, a_p \right] a_q e^{-ip \cdot x} - b_q^\dagger \left[ b_{\mathbf{q}}^\dagger, b_p \right] e^{ip \cdot x} \right\} \quad (16)$$

$$= \int \frac{d^3 p \, d^3 q}{(2\pi)^{3/2} \sqrt{2E_p}} \left\{ \left[ a_p, a_{\mathbf{q}}^\dagger \right] a_q e^{-ip \cdot x} - b_q^\dagger \left[ b_{\mathbf{q}}^\dagger, b_p \right] e^{ip \cdot x} \right\} \quad (17)$$

$$= \int \frac{d^3 p \, d^3 q}{(2\pi)^{3/2} \sqrt{2E_p}} \left\{ -\delta (\mathbf{p} - \mathbf{q}) a_q e^{-ip \cdot x} - b_q^\dagger \delta (\mathbf{p} - \mathbf{q}) e^{ip \cdot x} \right\} \quad (18)$$

$$= -\int \frac{d^3 p}{(2\pi)^{3/2} \sqrt{2E_p}} \left( a_p e^{-ip \cdot x} + b_p^\dagger e^{ip \cdot x} \right) \quad (19)$$

$$= -\psi \quad (20)$$