GREEN FUNCTION FOR INFINITE SQUARE WELL

The Green function for the Schrödinger equation has the form

\[ G^{+}(x,t_{x},y,t_{y}) = \theta(t_{x} - t_{y}) \sum_{n} \phi_{n}(x) \phi_{n}^{*}(y) e^{-iE_{n}(t_{x} - t_{y})} \]  

(1)

where \( \phi_{n} \) is an eigenfunction of the hamiltonian \( H \) with energy \( E_{n} \). By using various transformations, the Green function can be written in terms of other variables. In L&B’s Example 16.7 they show that in the momentum-time domain, we have

\[ G^{+}_{0}(p,t_{x},q,t_{y}) = \theta(t_{x} - t_{y}) \delta(p-q) e^{-iE_{p}(t_{x} - t_{y})} \]  

(2)

where \( q \) and \( p \) are the momenta in the initial and final states. The delta function conserves momentum so that we always have \( p = q \). As a result, sometimes the Green function is written in a simplified form by omitting the delta function:

\[ G^{+}_{0}(p,t_{x},t_{y}) = \theta(t_{x} - t_{y}) e^{-iE_{p}(t_{x} - t_{y})} \]  

(3)

In the case of a particle in the infinite square well of width \( a \), the energies are (with \( \hbar = 1 \)):

\[ E_{n} = \frac{n^{2} \pi^{2}}{2ma^{2}} \]  

(4)

The Green function in this case is therefore

\[ G^{+}_{0}(n,t_{x},t_{y}) = \theta(t_{x} - t_{y}) e^{-i\frac{n^{2} \pi^{2}}{2ma^{2}}(t_{x} - t_{y})} \]  

(5)

where we’ve written the momentum in the argument of \( G^{+}_{0} \) using the quantum number \( n \).

We can also write the Green function in the momentum-energy domain by doing a Fourier transform to eliminate the time, as is done in L&B’s Example 16.8. The result is
Green function for infinite square well

\[ G_0^+(p, E) = \frac{i}{E - E_p + i\epsilon} \]  \hspace{1cm} (6)

where the infinitesimal quantity \( \epsilon \) was introduced to ensure converge of the integral in the Fourier transform. For the square well, we have

\[ G_0^+(p, E) = \frac{i}{E - \frac{n^2 \pi^2}{2ma^2} + i\epsilon} \]  \hspace{1cm} (7)

L&B use the symbol \( \omega \) which I assume is the energy of the particle, so with that notation we have

\[ G_0^+(n, \omega) = \frac{i}{\omega - \frac{n^2 \pi^2}{2ma^2} + i\epsilon} \]  \hspace{1cm} (8)