ACTION OF A SCALAR FIELD THEORY AS A MOMENTUM INTEGRAL

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 17.3.

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We'll look at a scalar field theory defined by the Lagrangian density:

$$\mathcal{L}=\frac{1}{2}\partial_{\mu}\phi(x)\partial^{\mu}(x)-\frac{m^{2}}{2}[\phi(x)]^{2} \tag{1}$$

The field operator $\phi(x)$ can also be written as the Fourier transform of a field operator in momentum space:

$$\phi(x)=\frac{1}{(2\pi)^{4}}\int d^{4}p\,e^{ip\cdot x}\tilde{\phi}(p) \tag{2}$$

In this form, the Lagrangian density is

$$\mathcal{L}=\frac{1}{2(2\pi)^{8}}\int d^{4}p\,d^{4}q\,e^{i(p+q)\cdot x}\left(-p_{\mu}q^{\mu}-m^{2}\right)\tilde{\phi}(p)\tilde{\phi}(q) \tag{3}$$

The action is the integral of the Lagrangian density over all space and time, so we have

$$S=\int d^{4}x\,\mathcal{L} \tag{4}$$

$$=\frac{1}{2(2\pi)^{8}}\int d^{4}x\int d^{4}p\,d^{4}q\,e^{i(p+q)\cdot x}\left(-p_{\mu}q^{\mu}-m^{2}\right)\tilde{\phi}(p)\tilde{\phi}(q) \tag{5}$$

The integral over spacetime gives us a delta function according to the formula

$$\delta(p+q)=\frac{1}{(2\pi)^{4}}\int d^{4}x\,e^{i(p+q)\cdot x} \tag{6}$$

We get

$$S=\frac{1}{2(2\pi)^{4}}\int d^{4}p\,d^{4}q\,\delta(p+q)\left(-p_{\mu}q^{\mu}-m^{2}\right)\tilde{\phi}(p)\tilde{\phi}(q) \tag{7}$$

The delta function thus sets $q=-p$ so we can do the integral over q to get

$$S = \frac{1}{2(2\pi)^4}\int d^4p\ \left(p_\mu p^\mu - m^2\right)\tilde{\phi}(p)\,\tilde{\phi}(-p) \tag{8}$$

$$= \frac{1}{(2\pi)^4}\int d^4p\ \left(\frac{p^2-m^2}{2}\right)\tilde{\phi}(p)\,\tilde{\phi}(-p) \tag{9}$$

L&B add at the end of the problem that we can read off the propagator by taking the inverse of the term in parentheses and multiplying by $i/2$. That is, we take the factor

$$\frac{p^2-m^2}{2} \tag{10}$$

invert it and multiply by $i/2$ (and add in an infinitesimal imaginary part to prevent blowups).

I'm not sure why this is true, but if we do this, we get

$$\tilde{G}(p) = \frac{i}{p^2-m^2+i\epsilon} \tag{11}$$

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