SIMPLE HARMONIC OSCILLATOR PROPAGATOR

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 17.4.

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We can use the technique developed for the Klein-Gordon propagator to find a propagator for a simple harmonic oscillator. We begin with the Lagrangian for the oscillator which is

$$L = T - V \tag{1}$$

$$= \frac{1}{2}m\dot{x}^2(t) - \frac{1}{2}m\omega_0^2 x^2(t) \tag{2}$$

where $x(t)$ is the displacement as a function of time, m is the mass and ω_0 is the frequency of the oscillator.

We can write x as a Fourier transform:

$$x(t) = \frac{1}{2\pi}\int d\omega\, e^{-i\omega t}\tilde{x}(\omega) \tag{3}$$

The Lagrangian becomes

$$L = -\frac{m}{2(2\pi)^2}\int d\omega\, d\omega'\omega\omega' e^{i(\omega+\omega')t}\tilde{x}(\omega)\tilde{x}(\omega') - \frac{m\omega_0^2}{2(2\pi)^2}\int d\omega\, d\omega' e^{i(\omega+\omega')t}\tilde{x}(\omega)\tilde{x}(\omega') \tag{4}$$

The action S is the integral of L over time, so we have

$$S = \int L\, dt \tag{5}$$

$$= -\frac{m}{2(2\pi)}\int d\omega\, d\omega'\omega\omega'\tilde{x}(\omega)\tilde{x}(\omega')\int\frac{dt}{2\pi}\, e^{i(\omega+\omega')t} - \tag{6}$$

$$\frac{m\omega_0^2}{2(2\pi)}\int d\omega\, d\omega'\tilde{x}(\omega)\tilde{x}(\omega')\int\frac{dt}{2\pi}\, e^{i(\omega+\omega')t} \tag{7}$$

The integrals are delta functions, so we have

$$S = -\frac{m}{2(2\pi)}\int d\omega\, d\omega'\omega\omega'\tilde{x}(\omega)\tilde{x}(\omega')\delta(\omega+\omega') - \frac{m\omega_0^2}{2(2\pi)}\int d\omega\, d\omega'\tilde{x}(\omega)\tilde{x}(\omega')\delta(\omega+\omega') \tag{8}$$

$$= \frac{m}{2(2\pi)}\int d\omega\ \tilde{x}(\omega)\tilde{x}(-\omega)\left(\omega^2-\omega_0^2\right) \tag{9}$$

$$= \frac{1}{(2\pi)}\int d\omega\ \tilde{x}(\omega)\tilde{x}(-\omega)\frac{m}{2}\left(\omega^2-\omega_0^2\right) \tag{10}$$

Extracting the last factor from the integrand, inverting and multiplying by $i/2$ and adding in the infinitesimal imaginary part to prevent blowups gives us

$$\tilde{G}(\omega) = \frac{i}{m\left(\omega^2-\omega_0^2+i\epsilon\right)} \tag{11}$$

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