FEYNMAN DIAGRAMS FOR PHI-CUBED THEORY

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Reference: Tom Lancaster and Stephen J. Blundell, *Quantum Field Theory for the Gifted Amateur*, (Oxford University Press, 2014), Problem 19.2.

Mark Srednicki, *Quantum Field Theory*, Chapter 9.

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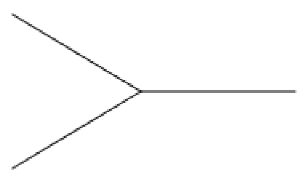
In ϕ^3 theory, the Lagrangian is

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\phi)^2-\frac{m^2}{2}\phi^2-\frac{\eta}{3!}\phi^3 \tag{1}$$

The interaction term is ϕ^3, which indicates that an interaction occurs when 3 particles coincide. Thus the interaction vertex diagram is



When we consider the expansion of the S-matrix for this interaction, we get the series

$$S=1+\sum_{n=1}^{\infty}\frac{(-i)^n}{n!}\int d^4x_1\int d^4x_2\ldots\int d^4x_n$$
$$T\left[\mathscr{H}_I(x_1)\mathscr{H}_I(x_2)\ldots\mathscr{H}_I(x_n)\right] \tag{2}$$

We then use Wick's theorem to evaluate the vacuum expectation values (VEV) of the terms in the series. This results in a sum of terms involving contractions of the operators. Since the VEV of any uncontracted operator is zero (because such an operator consists of a sum of creation operators (which act to the left giving zero when applied to the vacuum state $\langle 0|$) and annihilation operators (which act to the right, giving zero when applied to $|0\rangle$), only terms containing no left over uncontracted operators will give a non-zero contribution to the S-matrix expansion. Since the ϕ^3 interaction contains an odd number (3) of operators, only even-order terms in the S-matrix expansion will give non-zero contributions.

To find these terms, it's easiest to draw the corresponding Feynman diagrams. A second order diagram contains two vertices, each of which must consist of 3 lines joining at the vertex. Finding all the diagrams for a given

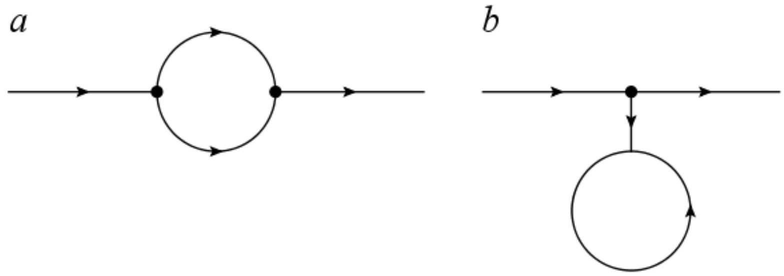


FIGURE 1. Second order single particle diagrams.

order requires a bit of imagination and I have to confess that I didn't get all of them on the first try. Chapter 9 in Srednicki's book gives a complete set of diagrams for ϕ^3 theory (although the theory he considers is a bit more complicated than that), so you can check your answers there. The second order diagrams are in Fig. 1.

The symmetry factors can be worked out using the rules given in L&B's Example 19.5. For Fig. 1a, we have two vertices connected by two lines, so $D = 2! = 2$ For Fig. 1b, the loop can be flipped so again we have $D = 2$.

Srednicki calls the symmetry factor S, and he also considers permutations of the incoming and outgoing particles, which L&B don't, so Srednicki's S is larger by a factor of 2.

Third order contributions are all zero since this involves 9 operators, which is an odd number. Fourth order diagrams must each have four vertices. The possible diagrams are in Fig. 2

Symmetry factors can be worked out for each diagram, but I won't go through all of them as they're not asked for in the question. In Fig. 2a, we have two pairs of vertices each connected by two internal lines, so we have $D = 2 \times 2 = 4$. For Fig. 2h, each loop contributes a factor of 2, and the two branches from the downward internal line can be swapped, giving another factor of 2, so we have $D = 2 \times 2 \times 2 = 8$. Similar considerations apply to the other diagrams.

Finally, we're asked to draw the fourth order diagrams for a scattering amplitude $\langle q_1 q_2 | S | p_1 p_2 \rangle$. In these diagrams, we have two incoming lines and two outgoing lines, and a total of four interaction vertices. The diagrams are in Fig. 3.

Each of these diagrams has a few permutations where we can swap the incoming and/or outgoing lines.

PINGBACKS

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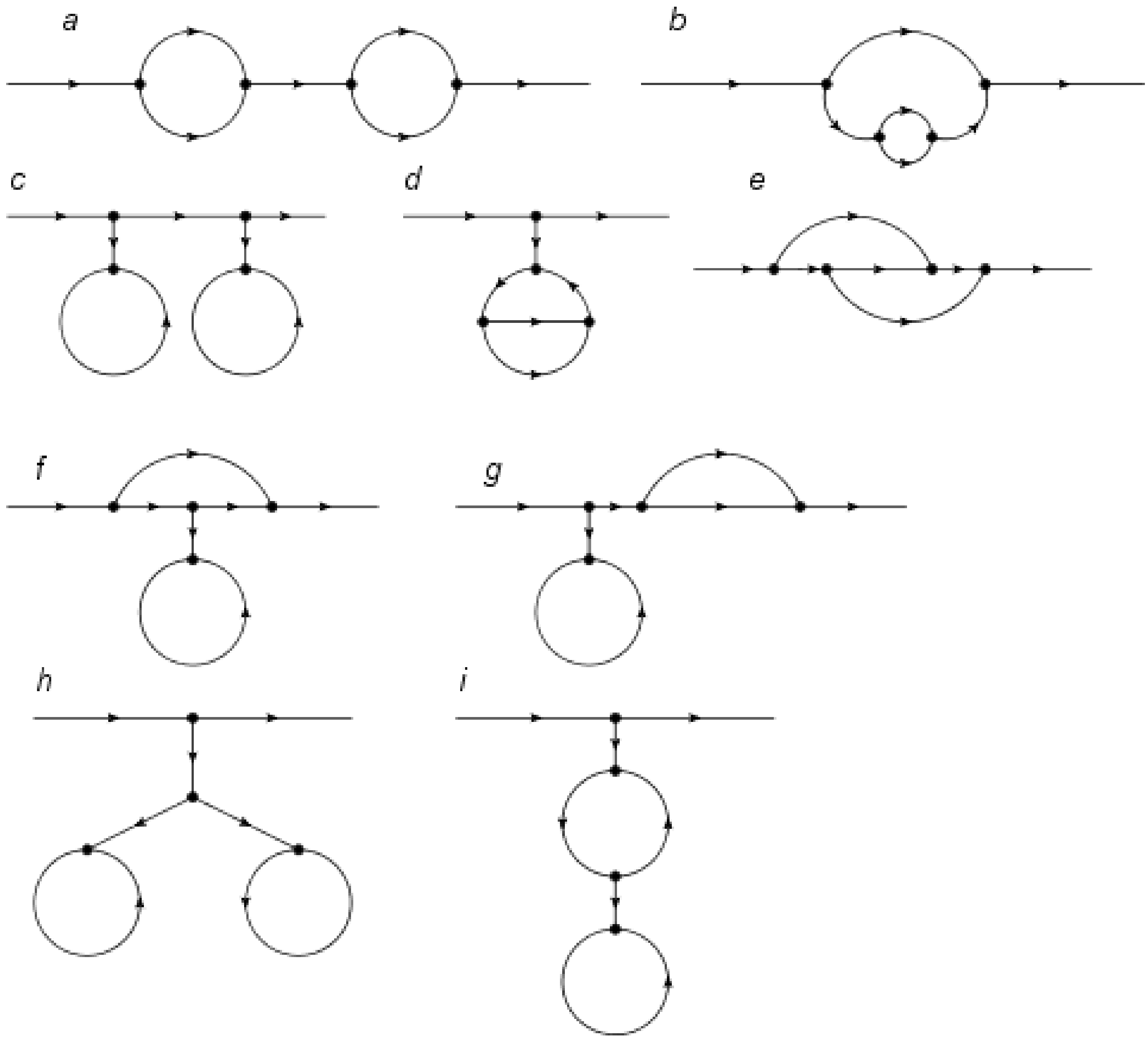

FIGURE 2. Fourth order single particle diagrams.

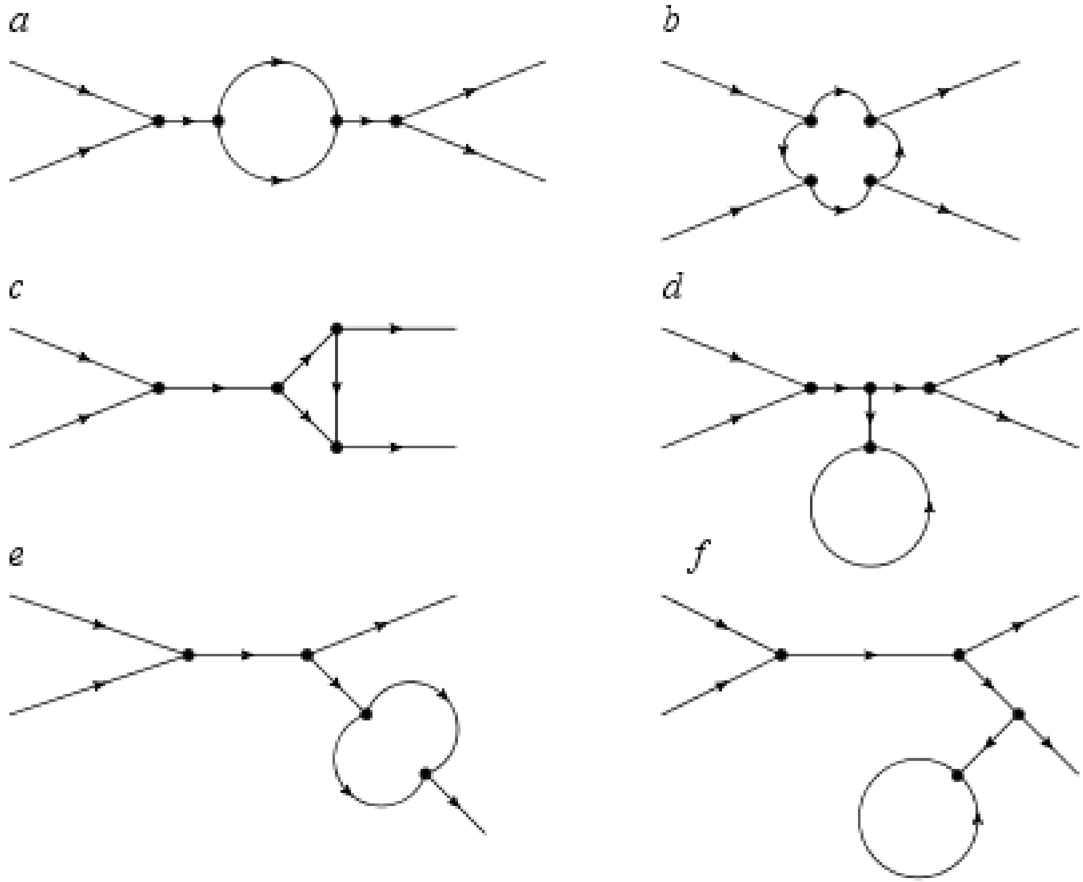

FIGURE 3. Fourth order scattering diagrams.